



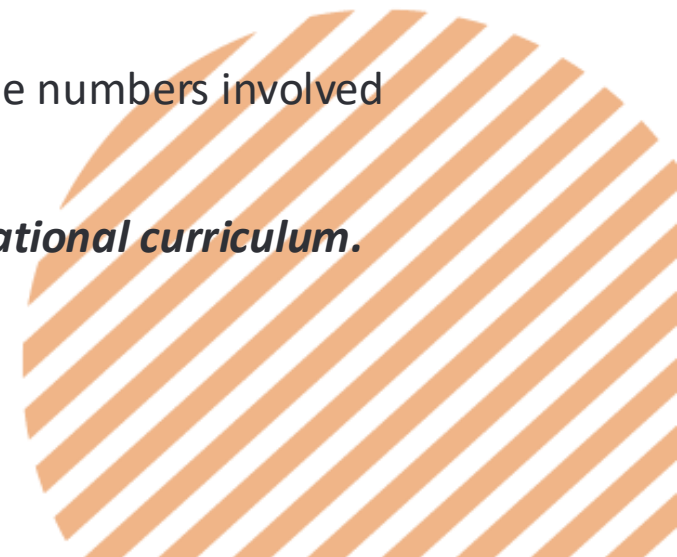
Progression in Mental and Written Strategies



Mental Calculations

- Pupils should learn a range of mental strategies to efficiently derive and recall number facts.
- The slides below are set out as guidance for a range of essential mental calculation strategies. They are not structured by domain and year group expectation but are often pre-requisite skills needed within written calculations.
- Schools may wish to use alternative manipulatives, dependant on available equipment and school preference.
- Pupils should be taught to be flexible by selecting a suitable strategy dependent on the numbers involved but also understand and be encouraged to use personal choice.

Please note that they highlight progression but are not necessarily matched with the national curriculum.



Written strategies

- **Age stage expectations:**

The written strategies outlined in the document are organised according to age stage expectations as set out in the National Curriculum 2014 and the method(s) shown for each year group should be modelled to the vast majority of pupils. However, it is vital that pupils are taught according to the pathway that they are currently working at and are showing to have 'mastered' a pathway before moving on to the next one.

- Schools may wish to use alternative manipulatives, dependent on available equipment and school preference. Ensure that when pupils move from base 10 to place value counters, they understand the unitary value e.g. a 10 counter represents 10 ones.

- **Choosing a calculation method:**

Before pupils opt for a written method, they should first consider these steps.

Can I do it in my head using a mental strategy?



Could I use some jottings to help me?



Should I use a formal written method to work it out?

Mental Calculations

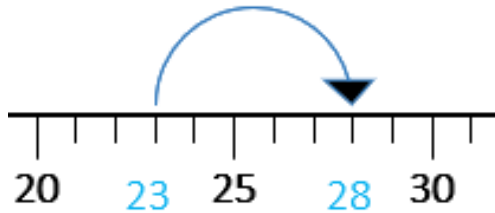
Adding a single-digit number to a 2-digit number

Using number bonds

$$23 + 5$$

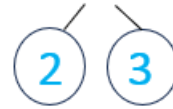
$$3 + 5 = 8$$

$$\text{so } 23 + 5 = 28$$



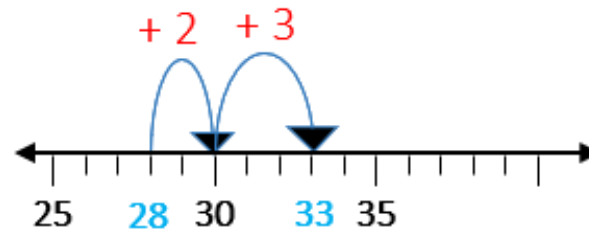
Using partitioning

$$28 + 5$$



$$28 + 5 = 28 + 2 + 3$$

$$28 + 5 = 33$$

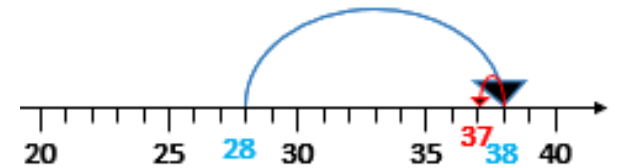


Using compensation

$$28 + 9$$

$$28 + 9 = 28 + 10 - 1$$

$$28 + 9 = 37$$



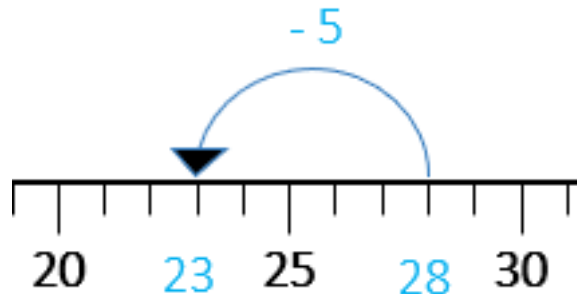
Subtracting a single-digit number from a 2-digit number

Using number bonds

$$28 - 5$$

$$8 - 5 = 3$$

$$\text{So } 28 - 5 = 23$$



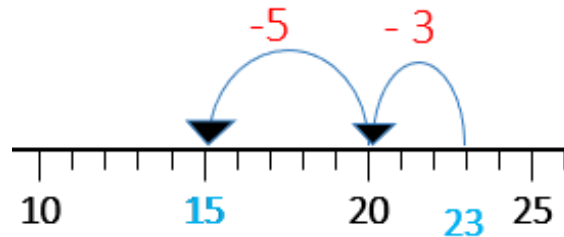
Using partitioning

$$23 - 8$$



$$23 - 8 = 23 - 3 - 5$$

$$23 - 8 = 15$$

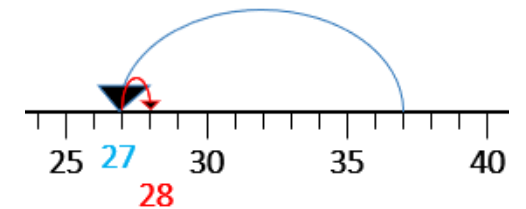


Using compensation

$$37 - 9$$

$$37 - 9 = 37 - 10 + 1$$

$$37 - 9 = 28$$

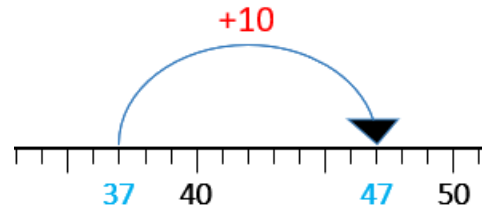


Adding two 2-digit numbers

Adding 10

$$37 + 10$$

$$37 + 10 = 47$$



Using partitioning with no bridging through a multiple of 10

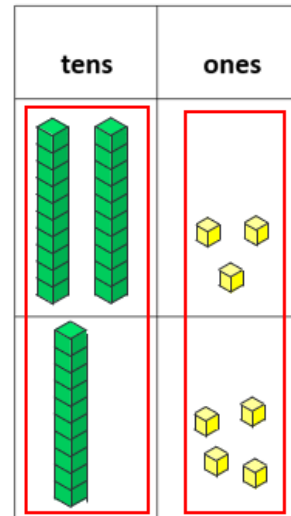
$$23 + 14$$

23 = 2 tens and 3 ones

14 = 1 ten and 4 ones

23 + 14 = 3 tens and 7 ones

$$23 + 14 = 37$$



Using partitioning, bridging through a multiple of ten

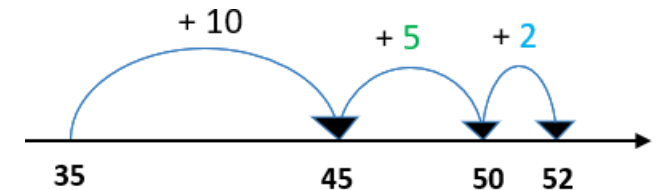
$$35 + 17$$

$$35 + 17 = 35 + 10 + 7$$

$$35 + 17 = 35 + 10 + 5 + 2$$

adds to make the next ten

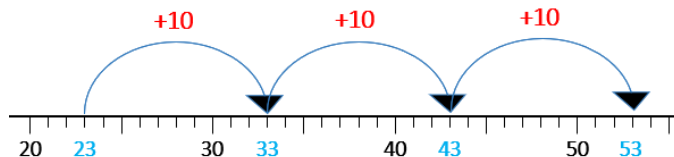
the other part of 7



Adding multiples of 10

$$23 + 30$$

$$23 + 30 = 53$$



Adding two 2-digit numbers

Using near doubles (consecutive)

$$45 + 46$$

$$45 + 46 = 45 + 45 + 1$$

$$45 + 46 = \text{double } 45 + 1$$

45 + 46		
45	46	1

$$45 + 46 = 90 + 1$$

$$45 + 46 = 91$$

Using near doubles (non consecutive)

$$25 + 27$$

$$25 + 27 = 25 + 25 + 2$$

$$25 + 27 = \text{double } 25 + 2$$

25 + 27		
25	25	2

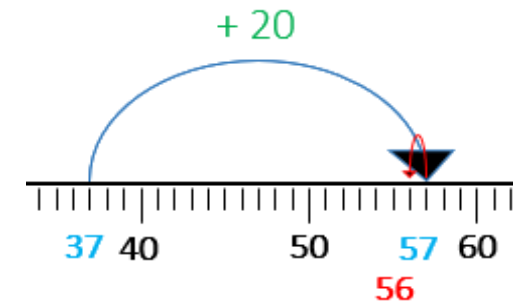
$$25 + 27 = 50 + 2$$

$$25 + 27 = 52$$

Using compensation

$$37 + 19$$

$$37 + 19 = 37 + 20 - 1$$

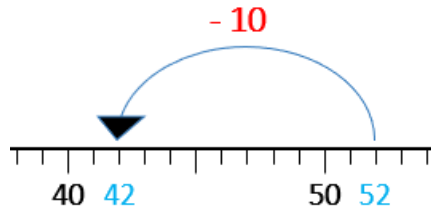


Subtracting two 2-digit numbers

Subtracting 10

$$52 - 10$$

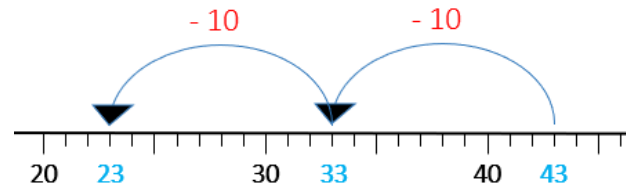
$$52 - 10 = 42$$



Subtracting multiples of 10

$$43 - 20$$

$$43 - 20 = 23$$



Using partitioning with no bridging through a multiple of 10

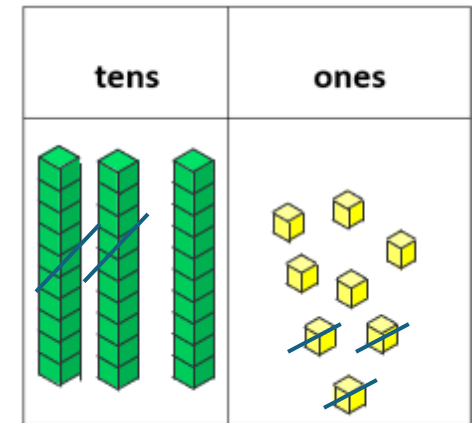
$$38 - 23$$

38 = 3 tens and 8 ones

23 = 2 tens and 3 ones

38 - 23 = 1 ten and 5 ones

$$38 - 23 = 15$$



Subtracting any two 2-digit numbers

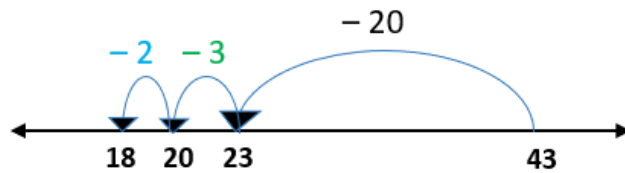
Using counting backwards, bridging through a multiple of ten

$$43 - 25$$

$$25 = 20 \text{ and } 5$$

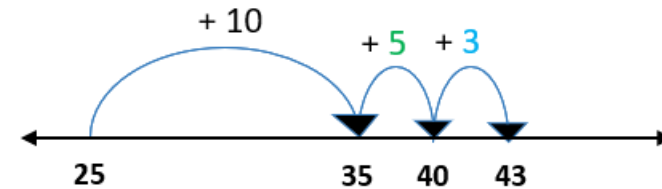
$$43 - 25 = 43 - 20 - 3 - 2$$

subtract to the multiple of ten
the other part of 5



Using counting on to find the difference

$$43 - 25$$



In total $10 + 5 + 3 = 18$ has been added to 25 to equal 43.

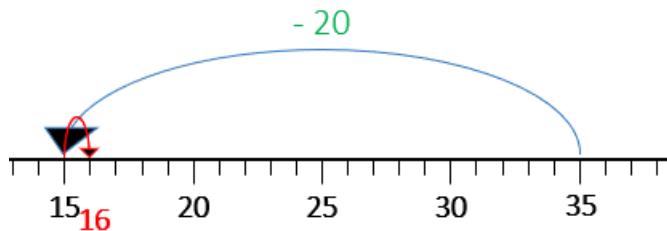
$$43 - 25 = 18$$

Subtracting two 2-digit numbers

Using compensation

$$35 - 19$$

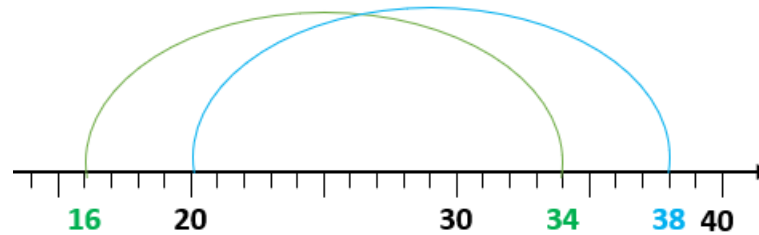
$$35 - 19 = 35 - 20 + 1$$



Using adjustment or constant difference

$$34 - 16$$

$34 - 16$ could be rewritten as $38 - 20$. The numbers have been adjusted to make the calculation simpler.



The difference between 34 and 16 is the same as the difference between 38 and 20 because 4 has been added to each number.

$$38 - 20 = 18$$

$$34 - 16 = 18$$

Adding numbers with decimals

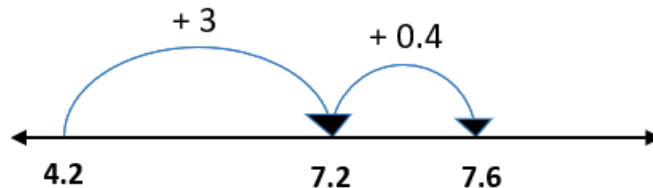
Using partitioning with no bridging through whole numbers

$$4.2 + 3.4$$

$$3.4 = 3 + 0.4$$

$$4.2 + 3.4 = 4.2 + 3 + 0.4$$

$$4.2 + 3.4 = 7.6$$



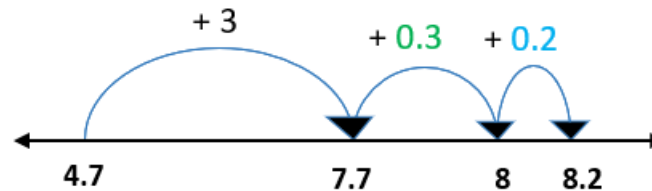
Using partitioning, bridging through whole numbers

$$4.7 + 3.5$$

$$3.5 = 3 + 0.5$$

$$4.7 + 3.5 = 4.7 + 3 + 0.3 + 0.2$$

adds to make the next whole
the other part of 0.5

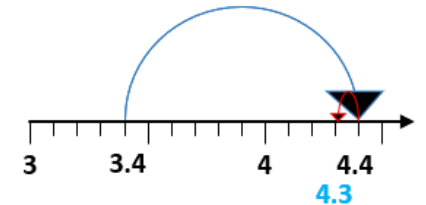


Using compensation

$$3.4 + 0.9$$

$$3.4 + 0.9 = 3.4 + 1 - 0.1$$

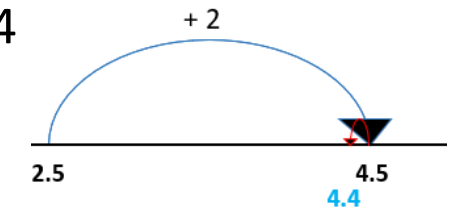
$$3.4 + 0.9 = 4.3$$



$$2.5 + 1.9$$

$$2.5 + 1.9 = 2.5 + 2 - 0.1$$

$$2.5 + 1.9 = 4.4$$



Adding three single-digit numbers

Using number bonds

$$\textcircled{8} + 6 + \textcircled{2}$$

$$8 + 6 + 2 = 8 + 2 + 6$$

$$8 + 6 + 2 = 10 + 6$$

$$8 + 6 + 2 = 16$$

Using doubles

$$\textcircled{3} + \textcircled{3} + 4$$

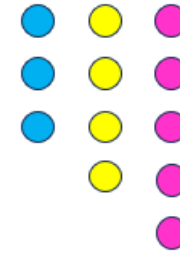
$$3 + 3 + 4 = \text{double } 3 + 4$$

$$3 + 3 + 4 = 6 + 4$$

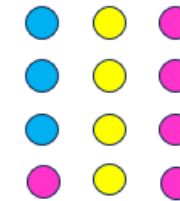
$$3 + 3 + 4 = 10$$

Using consecutive numbers

$$3 + 4 + 5$$



$$3 + 4 + 5 = 4 + 4 + 4$$



$$3 + 4 + 5 = 12$$

Multiplying by 10, 100 and 1,000 and multiples of 10

Multiplying a number by 10, 100 and 1,000

$$46 \times 10 = 460$$

H	T	O
	4	6
4	6	0

$$46 \times 100 = 4,600$$

Tth	Th	H	T	O
			4	6
	4	6	0	0

$$46 \times 1,000 = 46,000$$

Tth	Th	H	T	O
			4	6
4	6	0	0	0

Multiplying a number with decimals by 10, 100 and 1,000

$$0.34 \times 10 = 3.4$$

Tens	Ones	.	Tenths	Hundredths
	0	.	3	4
	3	.	4	

$$0.34 \times 100 = 34$$

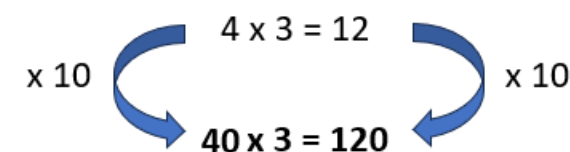
Hundreds	Tens	Ones	.	Tenths	Hundredths
		0	.	3	4
	3	4	.		

$$0.34 \times 1,000 = 340$$

Hundreds	Tens	Ones	.	Tenths	Hundredths
		0	.	3	4
3	4	0	.		

Multiplying a single-digit number by multiples of 10

$$40 \times 3$$



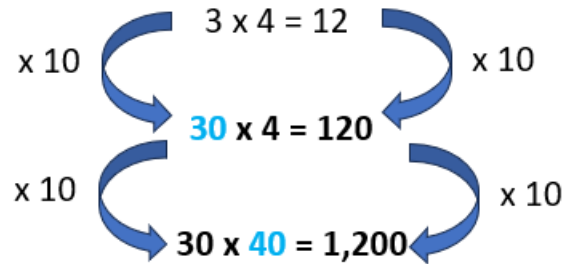
40×3 is 10 times bigger than 4×3 so the answer will be 10 times bigger

$$40 \times 3 = 120$$

Multiplying by multiples of 10 and 100

Multiplying multiples of 10 by multiples of 10

$$30 \times 40$$



Each step is 10 times bigger.
Each calculation is multiplied by 10, so the answer is becoming 10 times bigger each time.

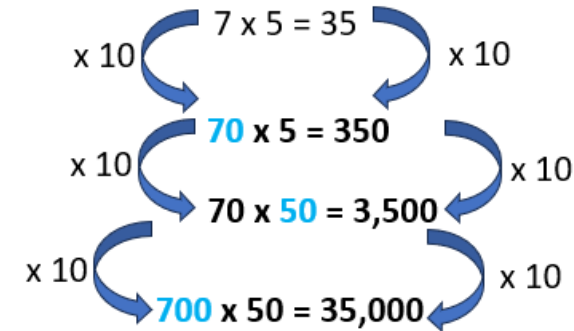
30×40 is 100 times bigger than 3×4

A diagram showing the relationship between $3 \times 4 = 12$ and $30 \times 40 = 1,200$. A green arrow labeled 'x 100' points from the first equation to the second.

$$30 \times 40 = 1,200$$

Multiplying multiples of 100 by multiples of 10

$$700 \times 50$$



700×50 is 1,000 times bigger than 7×5

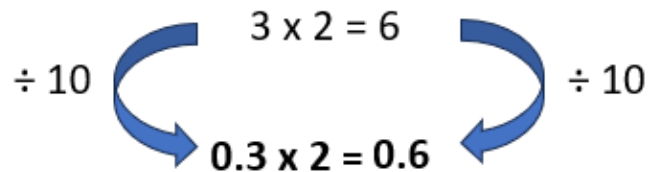
A diagram showing the relationship between $7 \times 5 = 35$ and $700 \times 50 = 35,000$. A green arrow labeled 'x 1000' points from the first equation to the second.

$$700 \times 50 = 35,000$$

Multiplying by tenths

Multiplying a single-digit number by tenths

$$0.3 \times 2$$

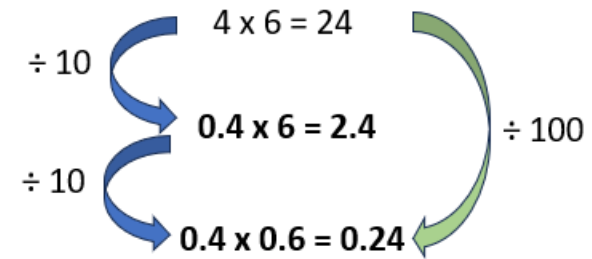


0.3×2 is 10 times smaller than 3×2 so the answer will be 10 times smaller

$$0.3 \times 2 = 0.6$$

Multiplying tenths by tenths

$$0.4 \times 0.6$$



0.4×6 is 10 times smaller than 4×6
so the answer will be 10 times smaller

0.4×0.6 is 10 times smaller than 0.4×6
so the answer will be 10 times smaller

0.4×0.6 is 100 times smaller than 4×6
 $0.4 \times 0.6 = 0.24$

Dividing by 10 and 100 and multiples of 10 by a single-digit number

Dividing a number by 10, 100 and 1,000

$$600 \div 10 = 60$$

Th	H	T	O
	6	0	0
		6	0

$$600 \div 100 = 6$$

Th	H	T	O
	6	0	0
			6

$$6,000 \div 1,000 = 6$$

Th	H	T	O
6	0	0	0
			6

Dividing a number with decimals by 10, 100 and 1,000

$$624 \div 10 = 62.4$$

Th	H	T	O	.	10th	100th
	6	2	4			
		6	2	.	4	

$$624 \div 100 = 6.24$$

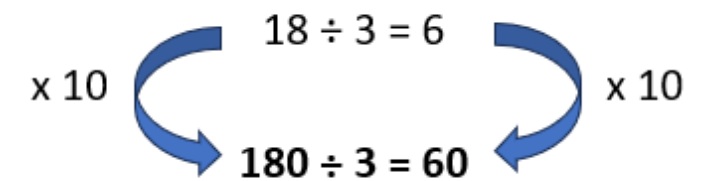
Th	H	T	O	.	10th	100th
	6	2	4			
			6	.	2	4

$$6,240 \div 1,000 = 6.24$$

Th	H	T	O	.	10th	100th
6	2	4	0			
			6	.	2	4

Identifying division facts

$$180 \div 3$$



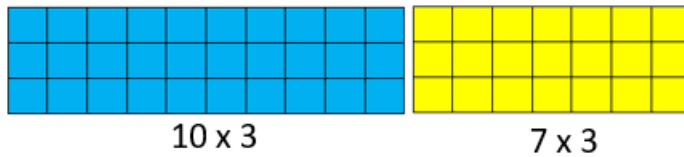
180 ÷ 3 is 10 times bigger than 18 ÷ 3 so the answer is 10 times bigger

$$180 \div 3 = 60$$

Multiplying 2-digits by a single-digit

Using partitioning

$$17 \times 3$$



$$17 \times 3 = (10 \times 3) + (7 \times 3)$$

$$17 \times 3 = 30 + 21 = 51$$

$$17 \times 3 = 51$$

Multiplying by 5

$$24 \times 5$$

$$24 \times 10 = 240$$

$$\text{Half of } 240 = 120$$

$$24 \times 5 = 120$$

To multiply a number by 5, multiply it by 10 and then halve it.

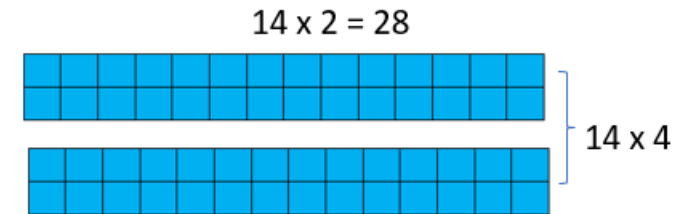
Using doubling

Multiplying a number by 4 is the same as doubling it, then doubling it again.

$$14 \times 4$$

$$14 \times 4 = 14 \times 2 \times 2$$

$$14 \times 4 = 56$$



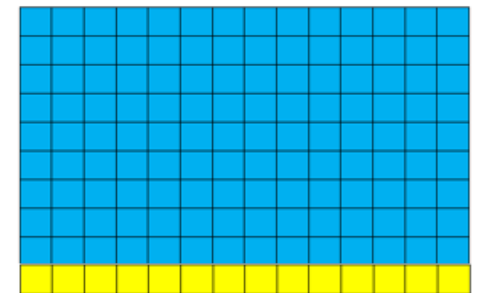
Multiplying by 9 using compensation

Instead of multiplying a number by 9, multiply it by 10 then subtract one group to be left with 9 groups.

$$14 \times 9$$

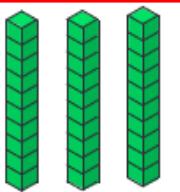
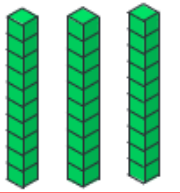
$$14 \times 9 = (14 \times 10) - 14$$

$$14 \times 9 = 126$$



Doubling

Doubling multiples of 10

tens	ones
	
	

Double 30

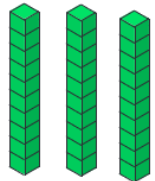

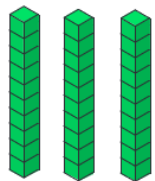

$30 = 3 \text{ tens}$

Double 3 = 6

Double 3 tens = 6 tens

Double 30 = 60

Doubling multiples of 5

tens	ones
	
	

Double 35

$35 = 30 + 5$

Double 30 = 60

Double 5 = 10

Double 35 = 60 + 10

Double 35 = 70

Doubling 2-digit numbers

Double 37

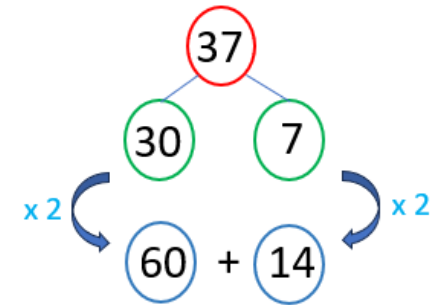
$37 = 30 + 7$

Double 30 = 60

Double 7 = 14

Double 37 = 60 + 14

Double 37 = 74



Doubling numbers with decimals

Double 4.7

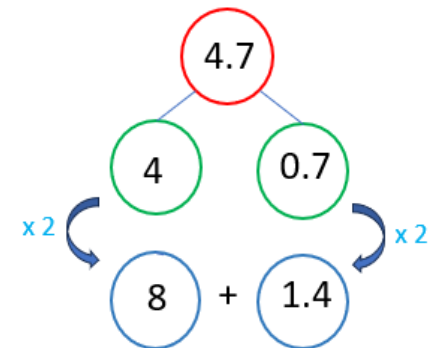
$4.7 = 4 + 0.7$

Double 4 = 8

Double 0.7 = 1.4

Double 4.7 = 8 + 1.4

Double 4.7 = 9.4



Halving

Halving multiples of 10 with an even number of tens

Half of 60

60 = 6 tens

$6 \div 2 = 3$

6 tens $\div 2 = 3$ tens

Half of 60 = 30

60	
30	30

Halving multiples of 10 with an odd number of tens

Half of 50

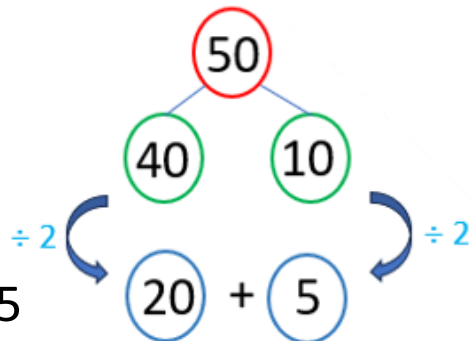
50 = 40 + 10

Half of 40 = 20

Half of 10 = 5

Half of 50 = 20 + 5

Half of 50 = 25



Halving 2-digit numbers

Half of 72

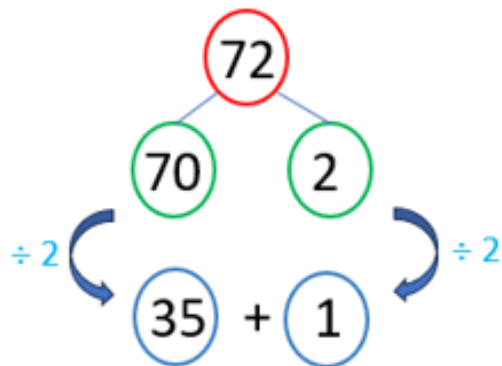
72 = 70 + 2

Half of 70 = 35

Half of 2 = 1

Half of 72 = 35 + 1

Half of 72 = 36



Halving 3-digit numbers

Half of 158

158 = 100 + 50 + 8

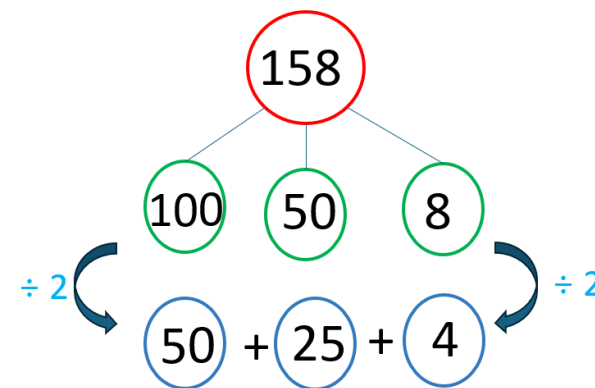
Half of 100 = 50

Half of 50 = 25

Half of 8 = 4

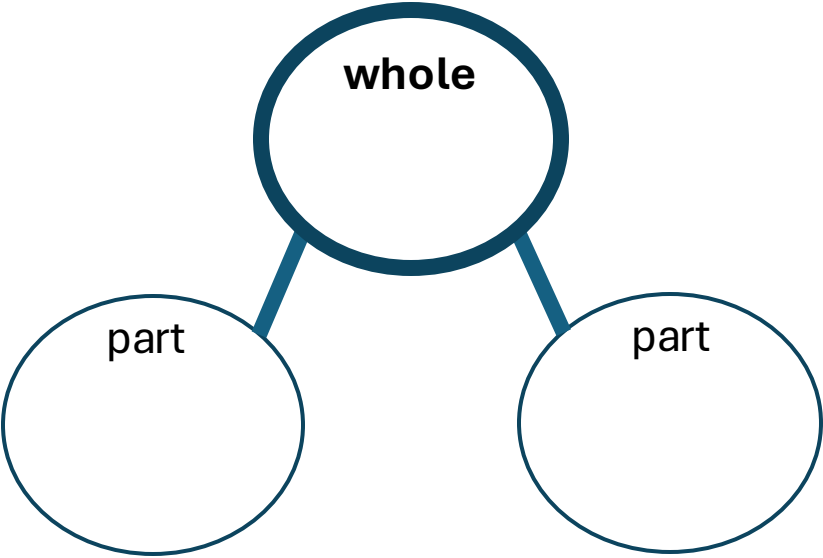
Half of 158 = 50 + 25 + 4

Half of 158 = 79



Written calculations: Addition and Subtraction

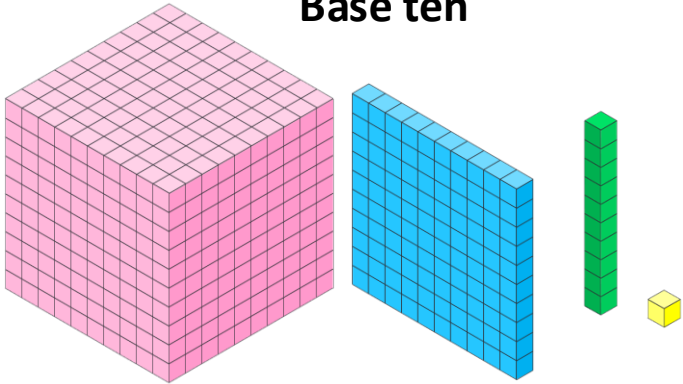
Part-whole models



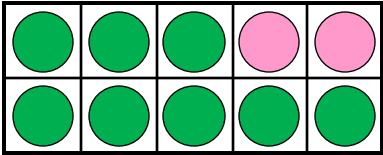
Place Value grid

thousands	hundreds	tens	ones

Base ten



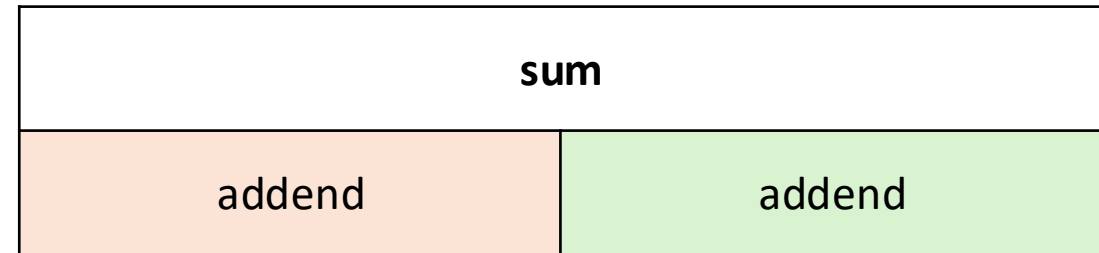
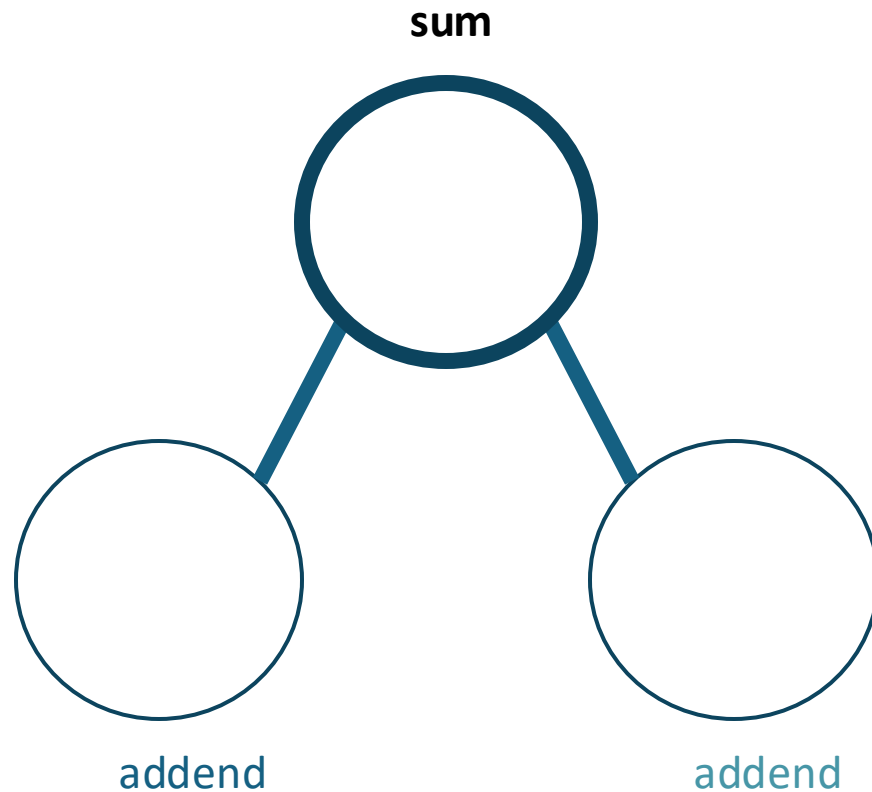
Tens Frame



Number line

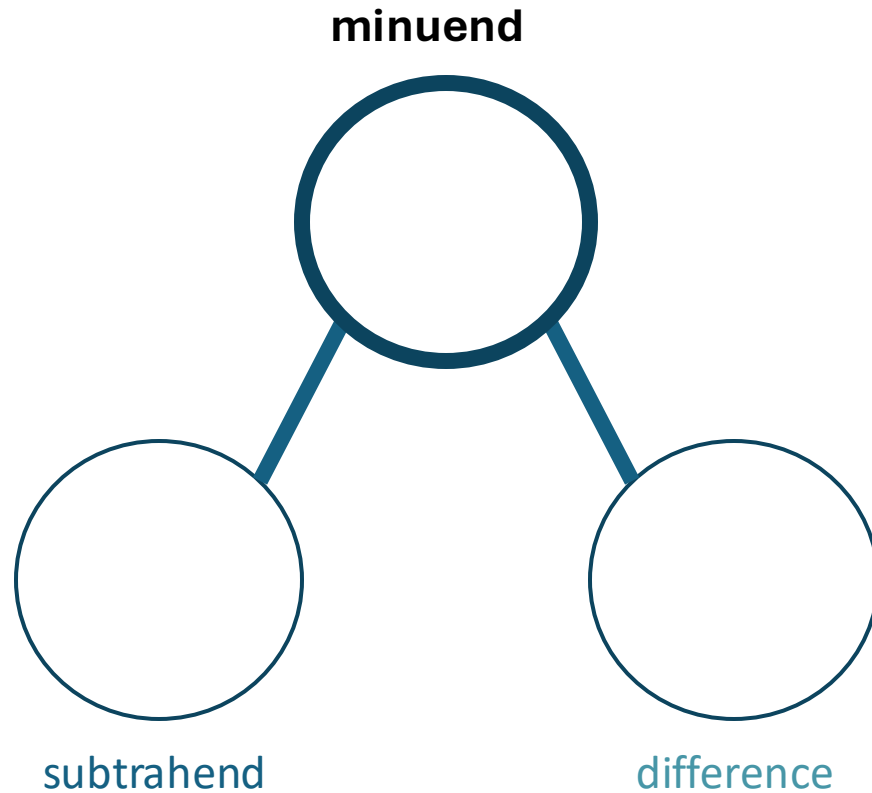


Additive Structure



$$\text{addend} + \text{addend} = \text{sum}$$

Additive Structure

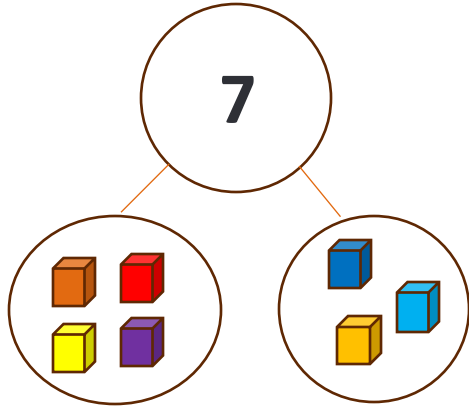


minuend	
subtrahend	difference

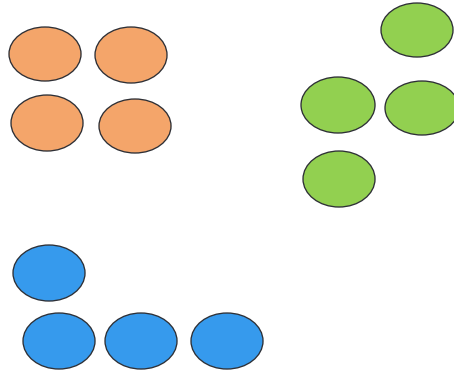
$$\text{minuend} - \text{subtrahend} = \text{difference}$$

Reception ELGs	
Addition and Subtraction	
Number	Numerical Patterns
<ul style="list-style-type: none"> • have a deep understanding of number to 10, including the composition of each number; • subitise (recognise quantities without counting) up to 5; • automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. 	<ul style="list-style-type: none"> • verbally count beyond 20, recognising the pattern of the counting system; • compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity; • explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

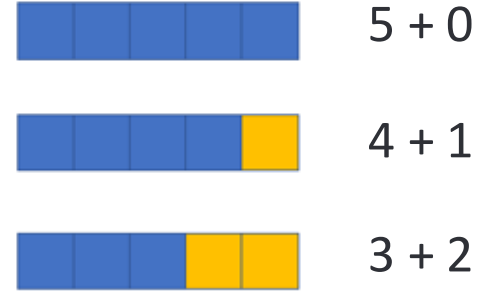
Composition of numbers



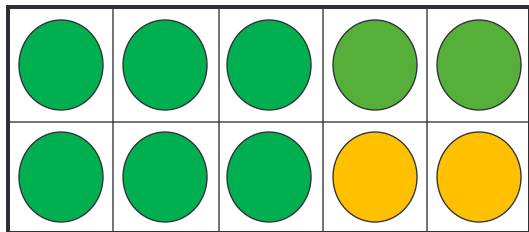
Subitise up to 5



Automatically recall number bonds up to 5



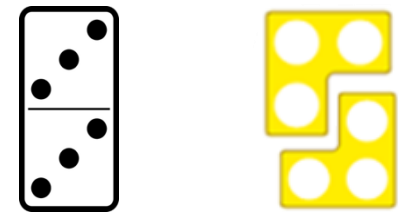
Explore and represent patterns within numbers up to 10



Explore and represent patterns with odds and evens



Doubling facts

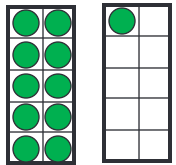
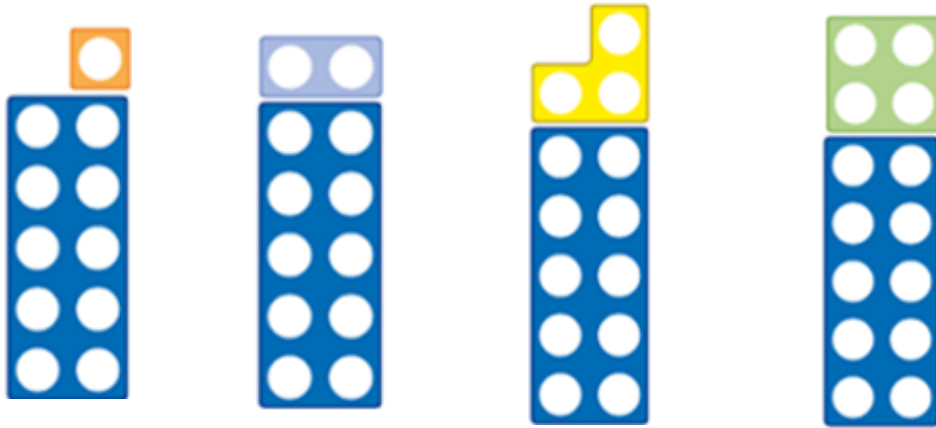


Year 1**Addition and Subtraction**

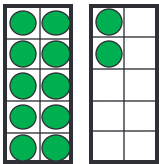
- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including 0
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = ? - 9$

Year 1 addition

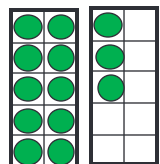
Understanding teen numbers as a 10 and ones



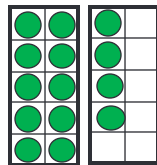
$$10 + 1 = 11$$



$$10 + 2 = 12$$



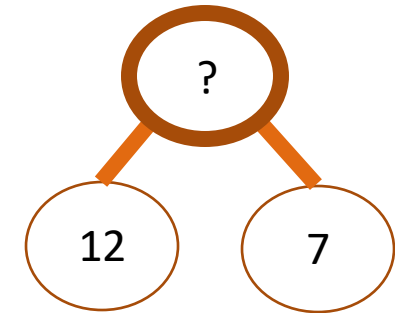
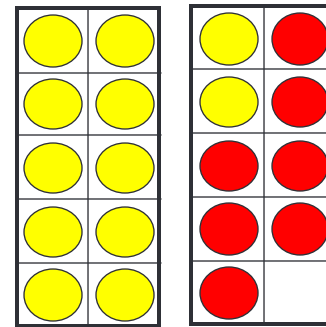
$$10 + 3 = 13$$



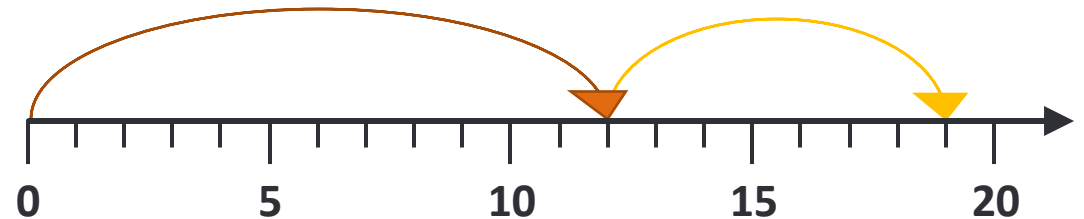
$$10 + 4 = 14$$

Can add a one-digit number to a two-digit number within 20

$$12 + 7 = 19$$



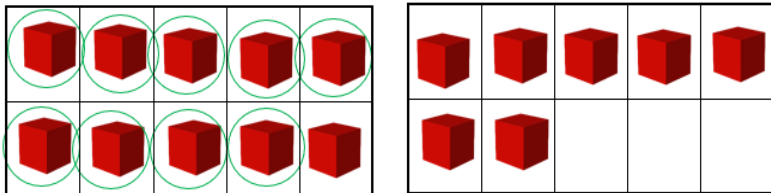
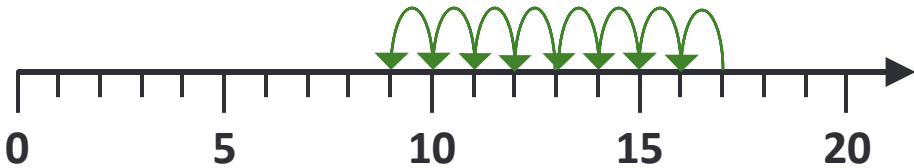
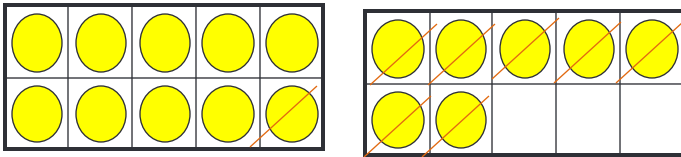
?	
12	7



Year 1 subtraction

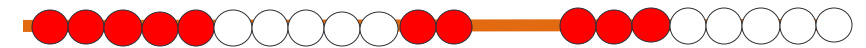
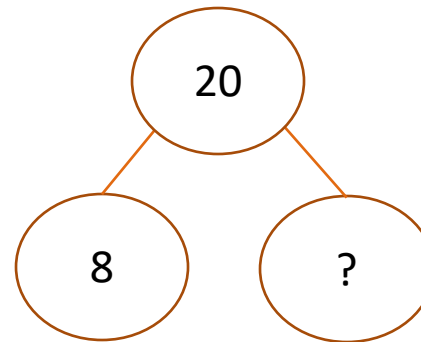
Taking away and counting back (used only for subtracting small numbers)

$$17 - 8$$

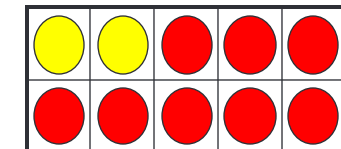
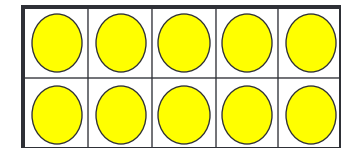


Can subtract a one-digit number from a two-digit number

$$20 - 8$$



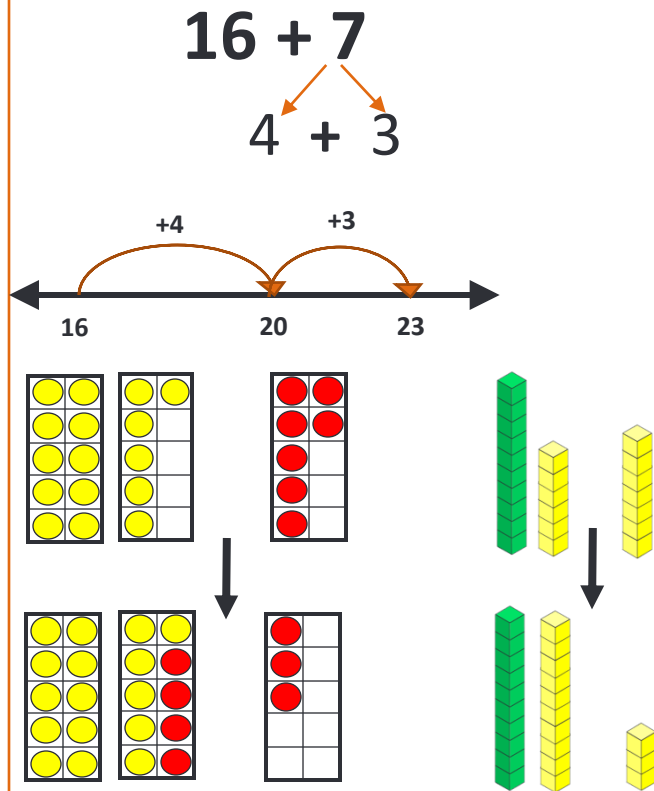
20	
8	?



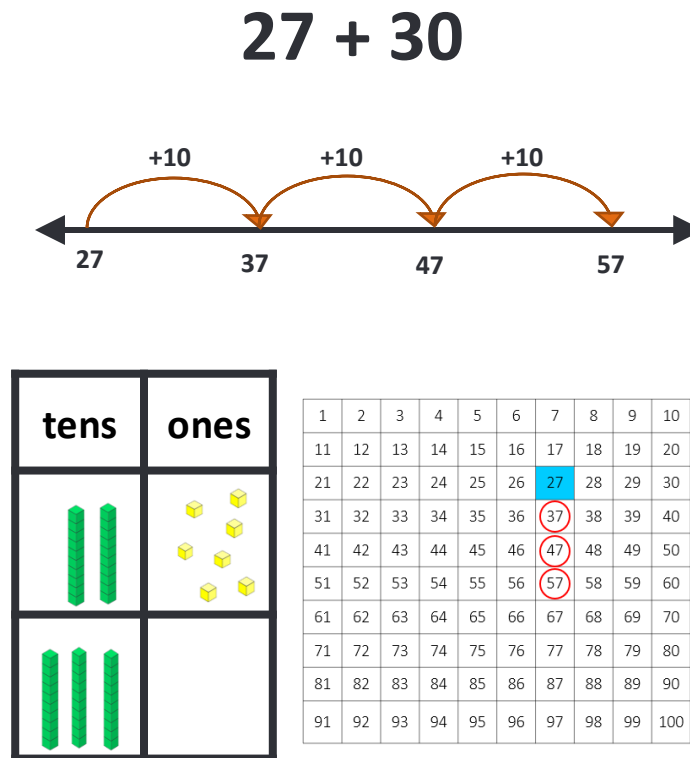
Year 2**Addition and Subtraction**

- solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s; a two-digit number and 10s; 2 two-digit numbers; adding 3 one-digit numbers
- show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

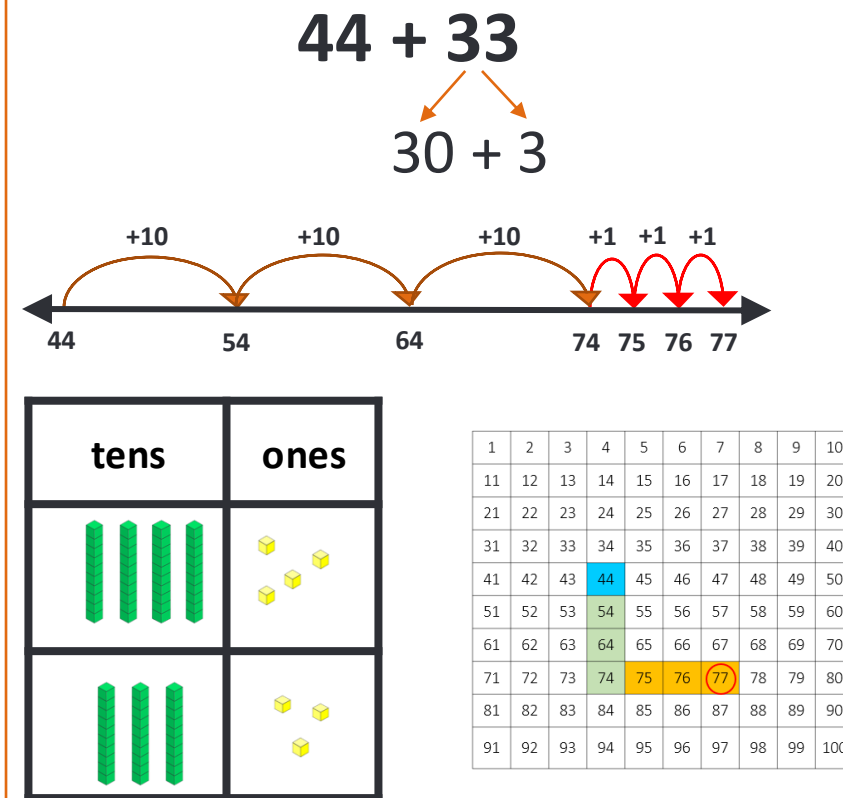
Can add a two-digit number and 1s using concrete objects or pictorial representations



Can add a two-digit number and 10s using concrete objects or pictorial representations



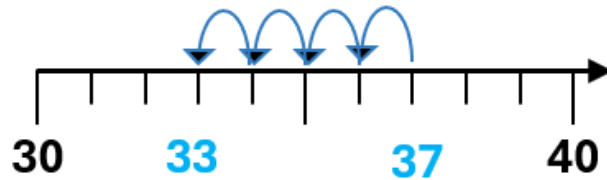
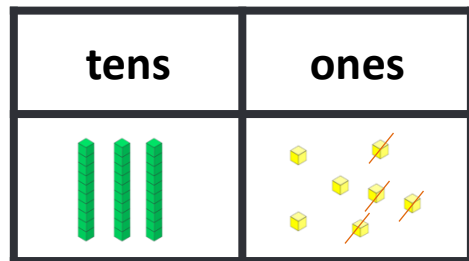
Can add 2 two-digit numbers using concrete objects or pictorial representations



Year 2 subtraction

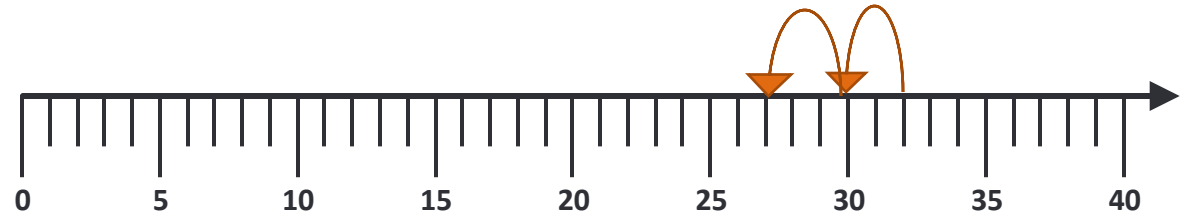
Can subtract a two-digit number and 1s using concrete objects or pictorial representations

$$37 - 4$$



Can subtract a two-digit number and 1s using concrete objects or pictorial representations

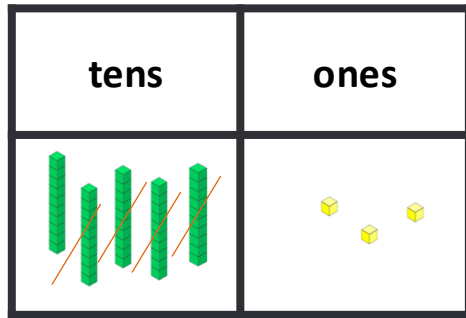
$$\begin{array}{r} 32 - 5 \\ \quad \swarrow \searrow \\ - 2 \quad - 3 \end{array}$$



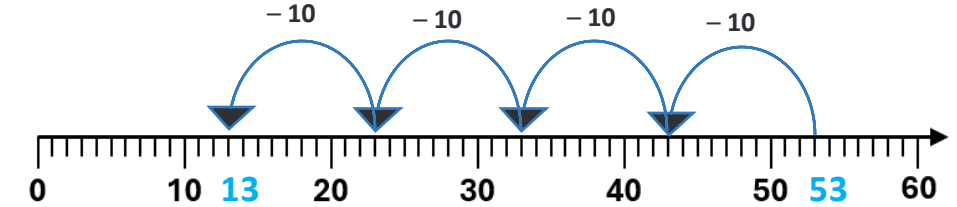
Year 2 subtraction

Can subtract a two-digit number and 10s using concrete objects or pictorial representations

$$53 - 40$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



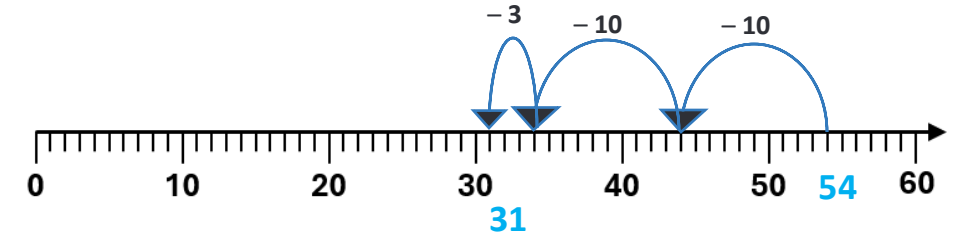
Can subtract a two-digit number using concrete objects or pictorial representations

$$\begin{array}{r} 54 \\ - 23 \\ \hline \end{array}$$

↙ ↘

$$\begin{array}{r} - 20 \\ - 3 \end{array}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



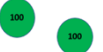
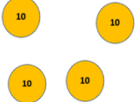
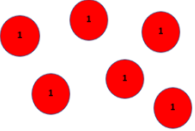
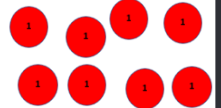
Year 3**Addition and Subtraction**

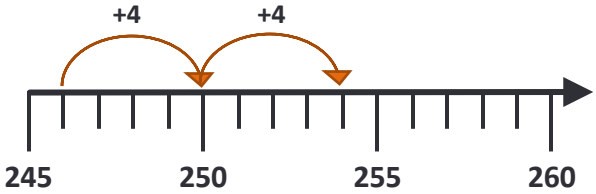
- add and subtract numbers mentally, including:
a three-digit number and 1s; a three-digit number and 10s; a three-digit number and 100s
- add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Year 3 addition

Can add a three-digit number and 1s mentally using partitioning


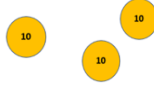
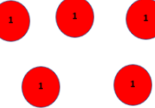
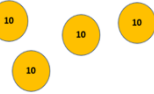
$246 + 8$
 $+ 4 \quad + 4$

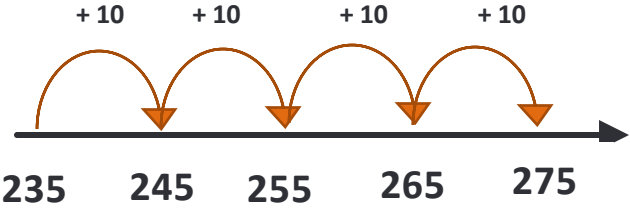
hundreds	tens	ones
		
		



Can add a three-digit number and 10s

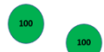
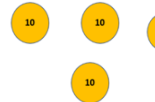
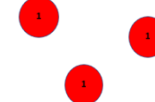

$235 + 40$

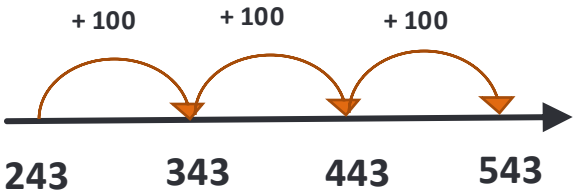
hundreds	tens	ones
		
		



Can add a three-digit number and 100s

$243 + 300$

hundreds	tens	ones
		
		



Year 3 addition

Can add numbers with two digits using the expanded column method (no regrouping)

$$42 + 31$$

tens	ones
10 10 10 10	1 1
10 10 10	1

4	0	+	2
3	0	+	1
7	0	+	3

Can add numbers with two digits using the expanded column method (regrouping)

$$47 + 36$$

tens	ones
10 10 10 10	1 1 1 1 1 1 1 1 1
10 10 10	1 1 1 1 1 1 1 1 1

10

4	0	+	7
3	0	+	6
8	0	+	3

10

Can add numbers with up to three digits using the expanded column method

$$367 + 185$$

hundreds	tens	ones
100 100 100	0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1 1 1
100	0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1

100 10

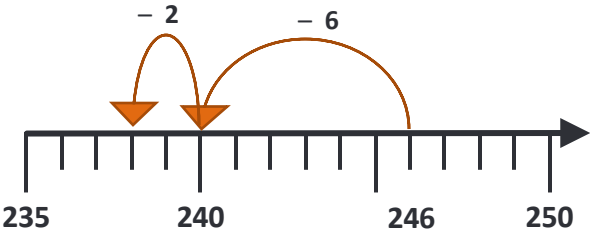
3	0	0	+	6	0	+	7
1	0	0	+	8	0	+	5
5	0	0	+	5	0	+	2

100 10

Year 3 subtraction

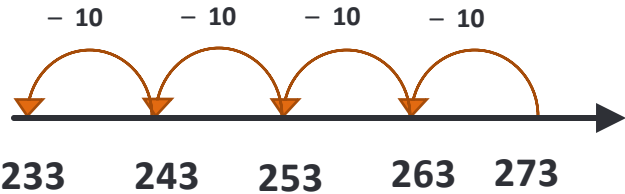
Can subtract a three-digit number and 1s mentally using decomposition (bridging ten)

$$\begin{array}{r}
 246 - 8 \\
 - 6 - 2
 \end{array}$$



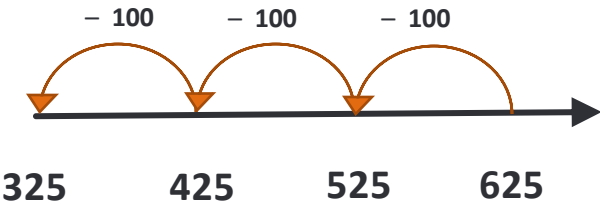
Can subtract a three-digit number and 10s

$$273 - 40$$



Can subtract a three-digit number and 100s

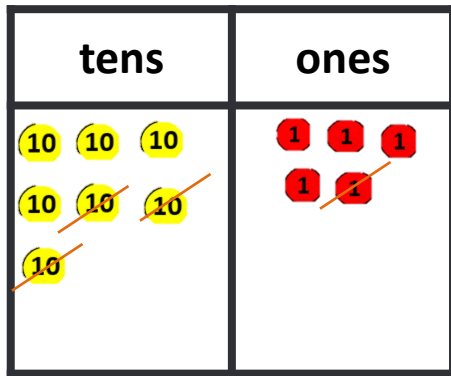
$$625 - 300$$



Year 3 subtraction

Can subtract numbers with two digits using the expanded column method (no exchange)

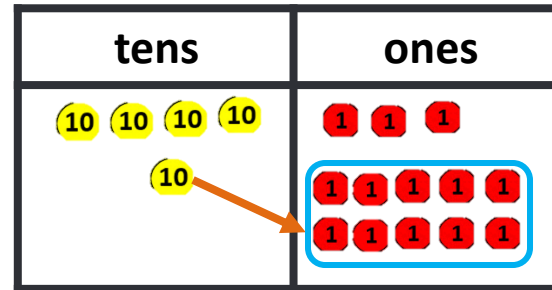
$$75 - 31$$



7	0	+	5
3	0	+	1
4	0	+	4

Can subtract numbers with two digits using the expanded column method (with exchange)

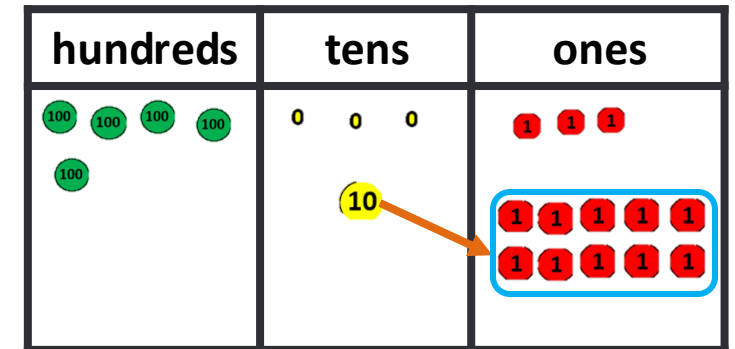
$$53 - 35$$



5	0	+	3
3	0	+	5
1	0	+	8

Can subtract numbers with up to three digits using the expanded column method (with exchange)

$$543 - 324$$



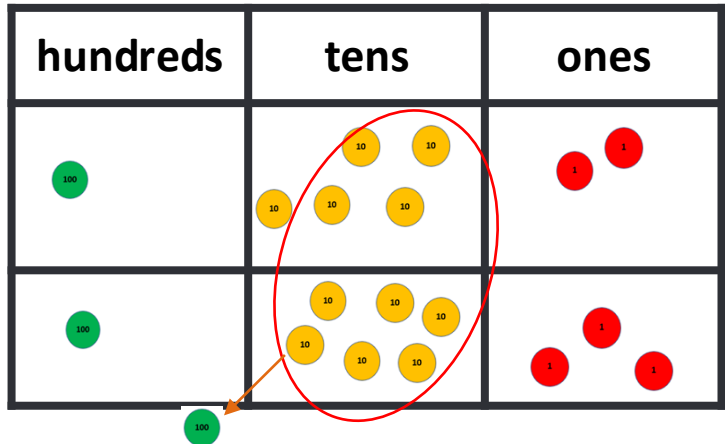
5	0	0	+	4	0	+	3
3	0	0	+	2	0	+	4
2	0	0	+	1	0	+	9

Year 4**Addition and Subtraction**

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Year 4 addition

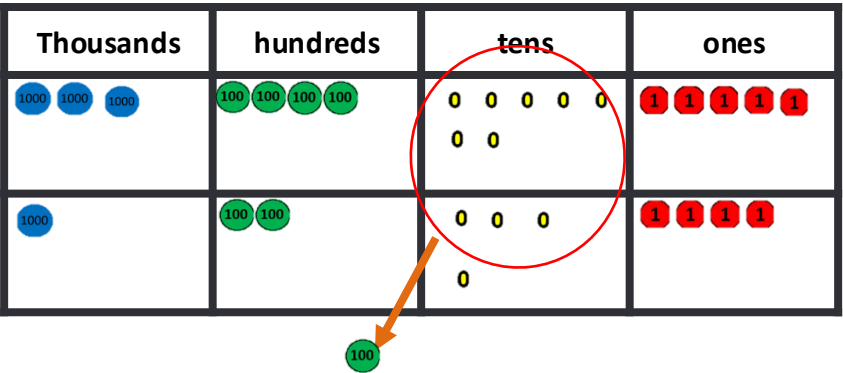
Can add three-digit numbers using the formal column addition method



$$152 + 163$$

1	5	2
1	6	3
3	1	5
1		

Can add four-digit numbers using the formal column addition method

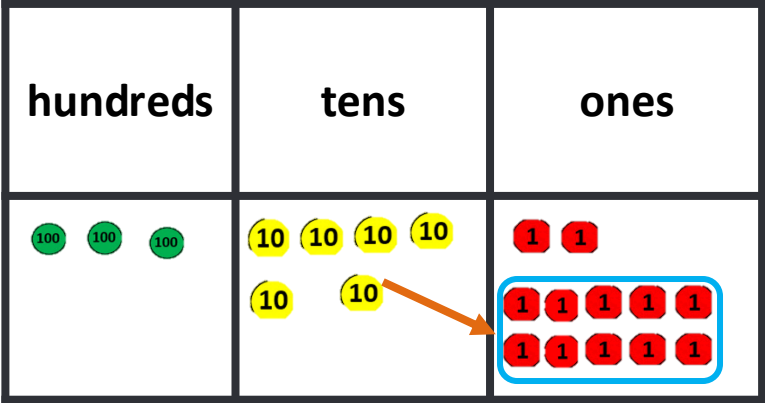


$$3,475 + 1,244$$

3	4	7	5
1	2	4	4
4	7	1	9
1			

Year 4 subtraction

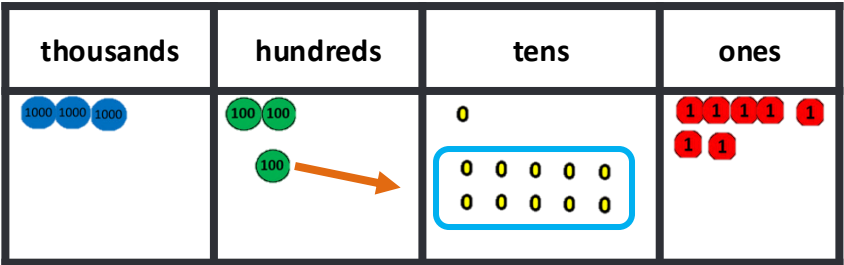
Can subtract three-digit numbers using the formal column subtraction method



$$362 - 124$$

3	6 ¹ 2	
1	2	4
2	3	8

Can subtract four-digit numbers using the formal column subtraction method



$$3,317 - 1,254$$

3	3 ¹ 1	7	
1	2	5	4
2	0	6	3

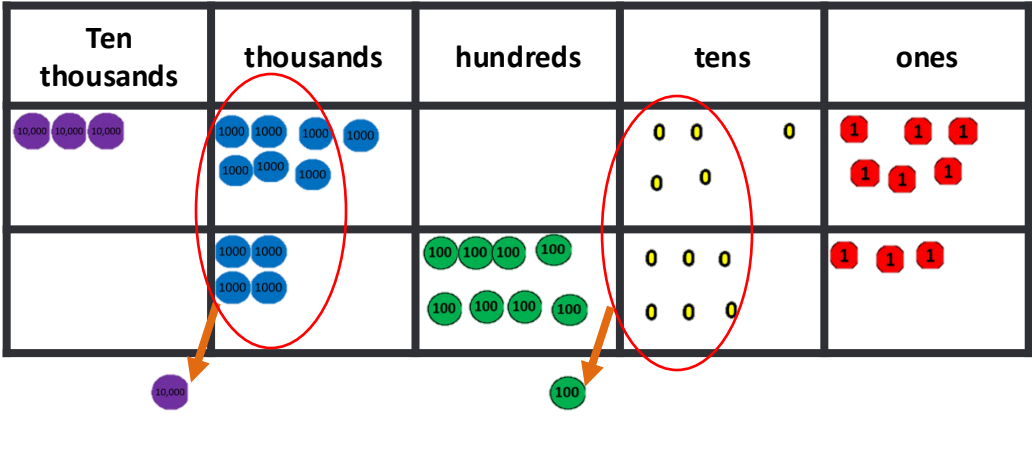
Year 5**Addition and Subtraction**

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving number up to three decimal places

Year 5 addition

Can add numbers with more than four-digits

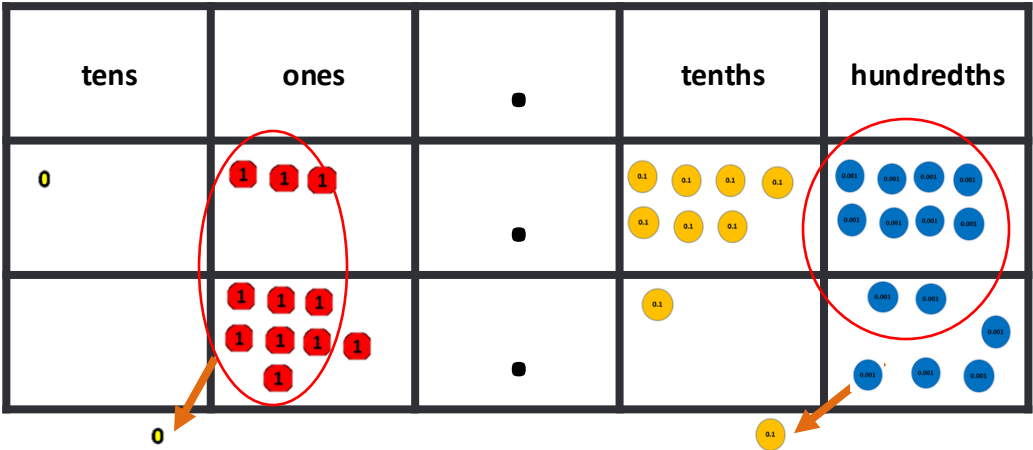
$$37,056 + 4,863$$



	3	7	0	5	6
+		4	8	6	3
	4	1	9	1	9
	1		1		

Can add numbers with wholes and decimals up to three decimal places

$$13.78 + 8.16$$

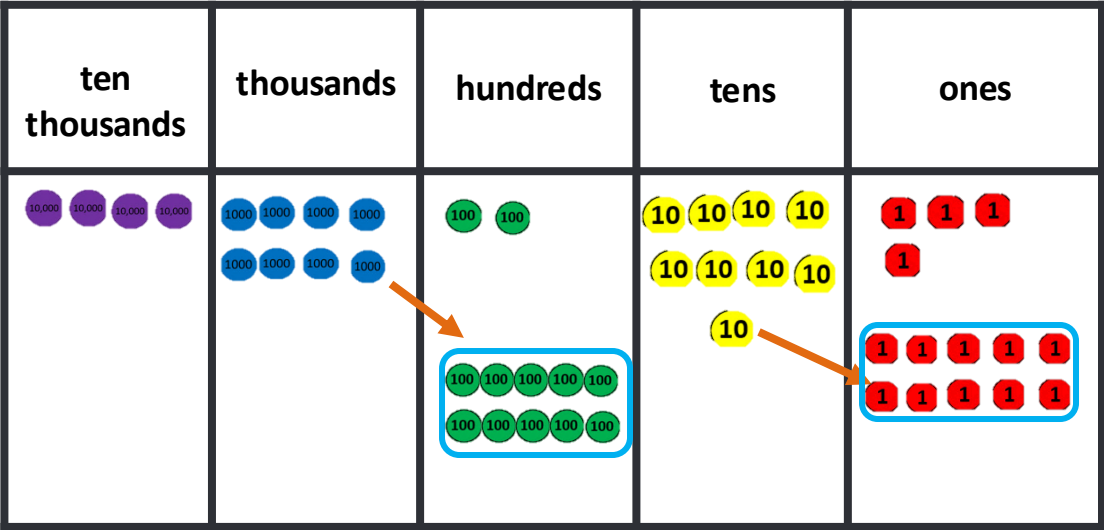


	1	3	.	7	8
+		8	.	1	6
	2	1	.	9	4
	1			1	

Year 5 subtraction

Can subtract numbers with more than four-digits

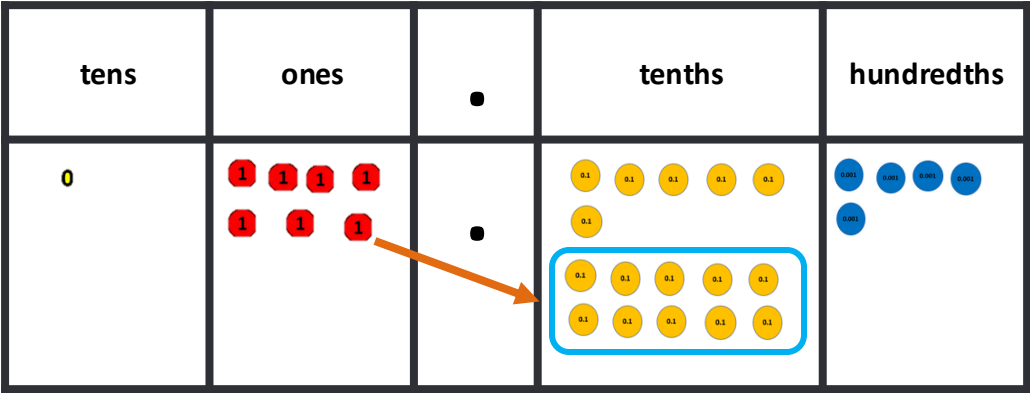
48,294 – 13,478



	4	8	2	9	4
–	1	3	4	7	8
	3	4	8	1	6

Can subtract numbers with wholes and decimals up to three decimal places

17.65 – 6.82



	1	7	.	6	5
–		6	.	8	2
	1	0	.	8	3

Year 6**Addition and Subtraction**

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Year 6 addition and subtraction

Can add numbers with wholes and decimals up to three decimal places with different numbers of digits

$$57.3 + 32.908$$

	5	7	.	3	0	0	
+	3	2	.	9	0	8	
	9	0	.	2	0	8	
	1	1					

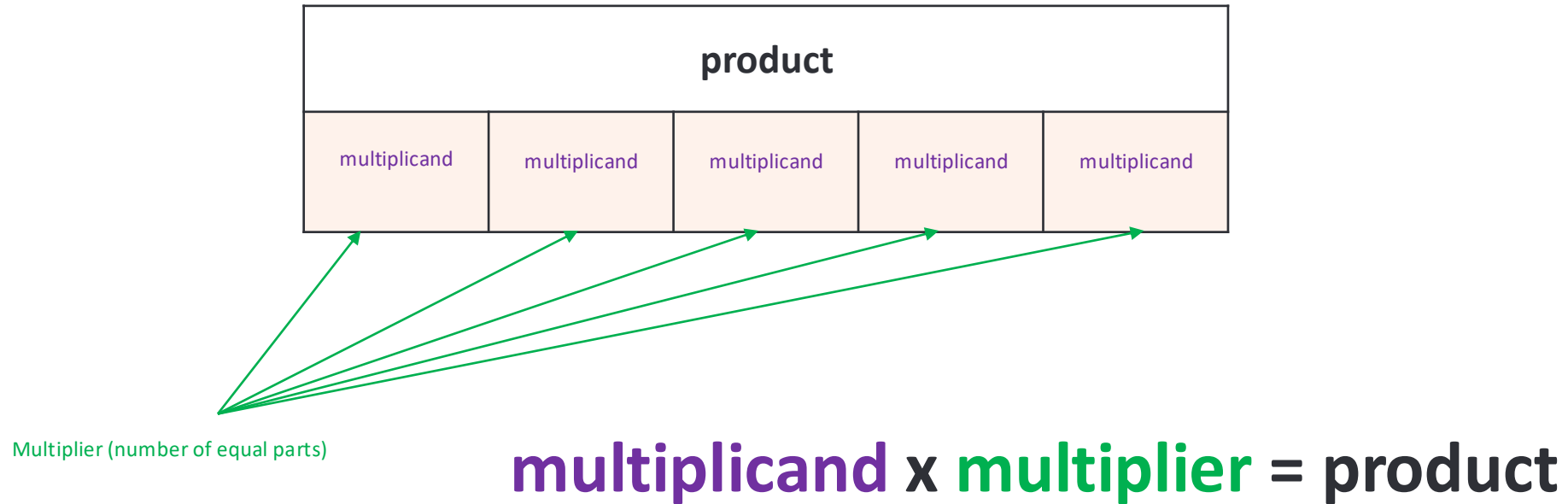
Can subtract numbers with wholes and decimals up to three decimal places with different numbers of digits

$$67.26 - 23.917$$

	6	7 ₆	.	¹ 2	6 ₅	¹ 0	
-	2	3	.	9	1	7	
	4	3	.	3	4	3	

Written calculations: Multiplication and Division

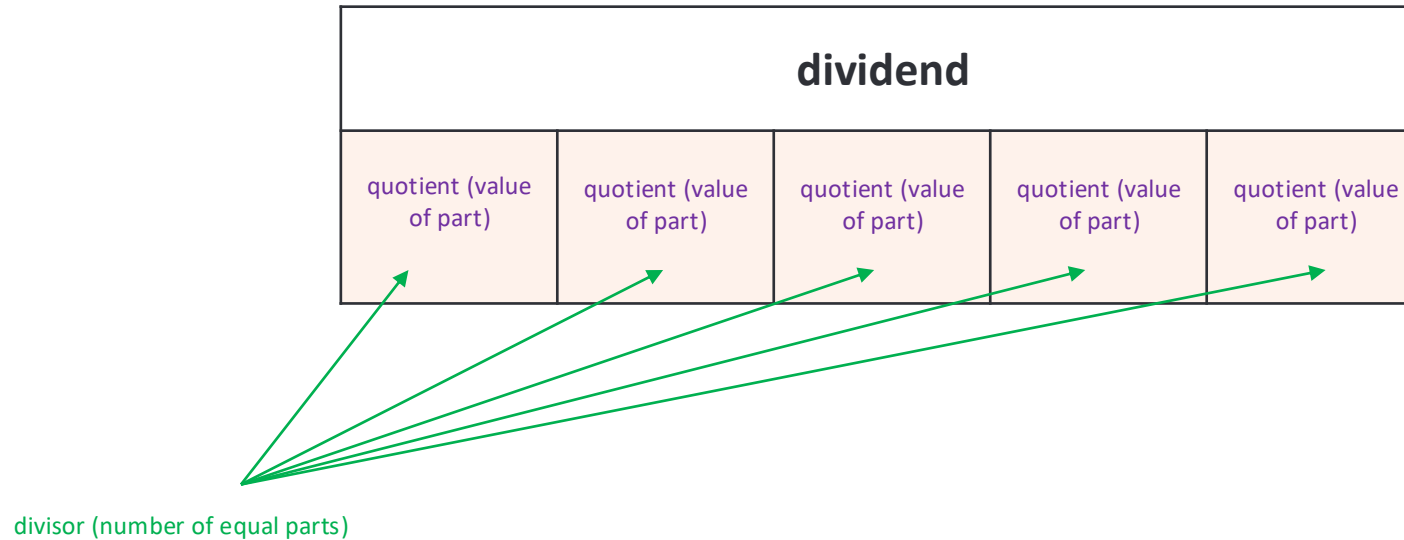
Multiplicative Structure



The **multiplicand** and the **multiplier** are both factors of the product.

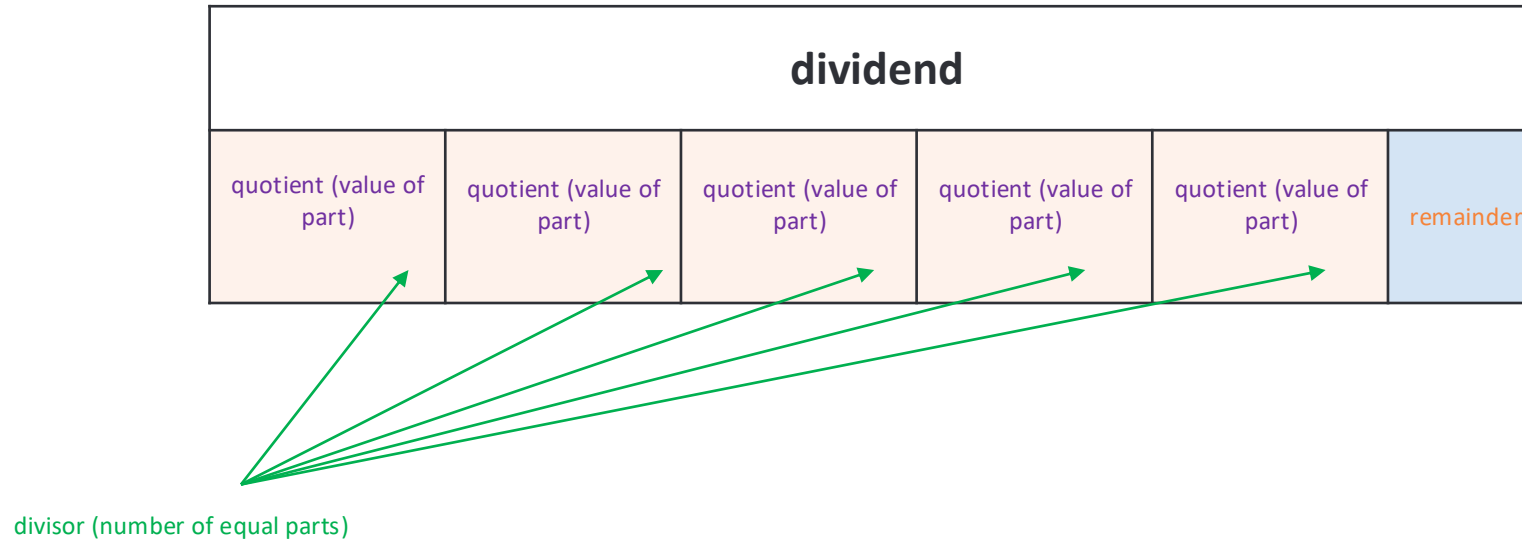
$$\text{factor} \times \text{factor} = \text{product}$$

Multiplicative Structure



$$\text{dividend} \div \text{divisor} = \text{quotient}$$

Multiplicative Structure

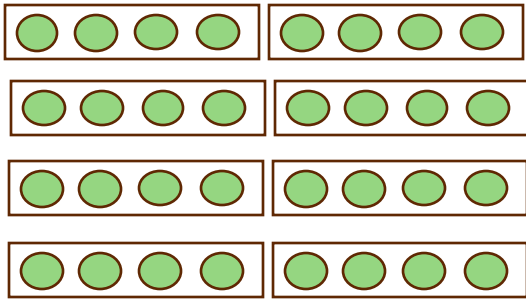


dividend ÷ **divisor** = quotient plus **a remainder**

Quotative and partitive division

Quotative division

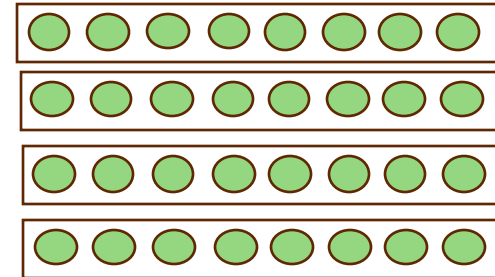
There are 32 pupils in a class.
They are put into groups of 4.



There are 4 pupils in each group.
There are 8 groups.

Partitive division

There are 32 pupils in a class.
They are put into 4 groups.



There are 4 groups.
There are 8 pupils in each group.

EYFS

Multiplication and Division

- explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally

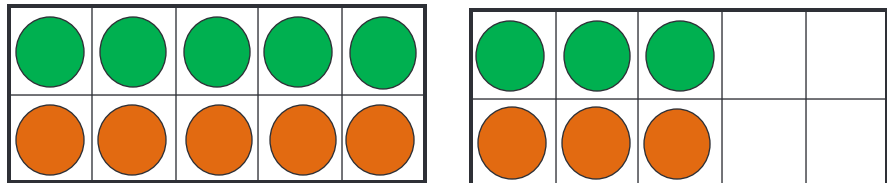
Reception ELGs

Can double numbers and quantities up to 10 using practical objects

Double 3  Double 3 is 6

Double 4  Double 4 is 8

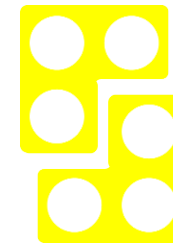
Double 5  Double 5 is 10



$$16 = 8 + 8$$

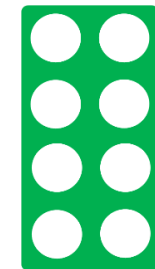
Can halve numbers and quantities up to 10 using practical objects with equal sharing

Half of 6



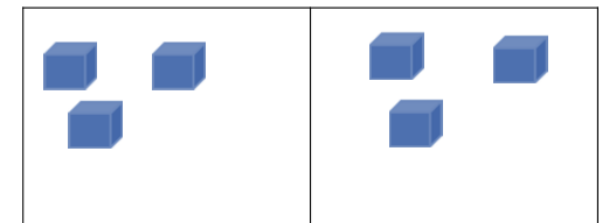
Half of 6 = 3

Half of 8



Half of 8 = 4

Half of 6 = 3



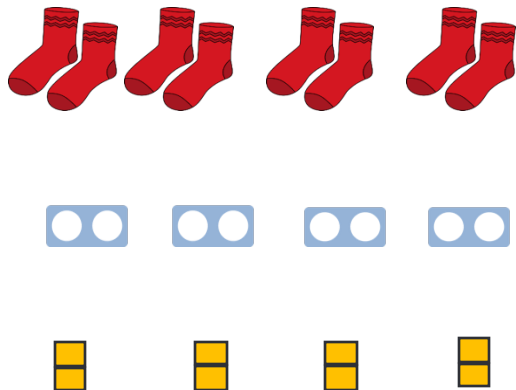
Year 1

Multiplication and Division

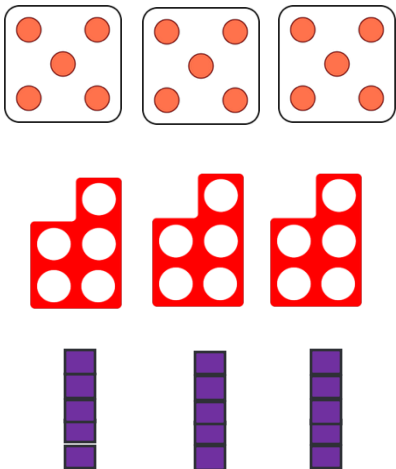
- count in multiples of twos, fives and tens
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Year 1 multiplication

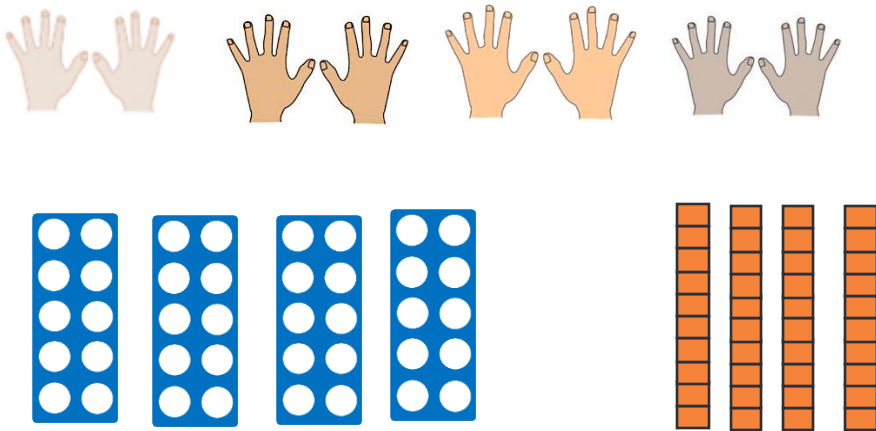
Can count in 2s



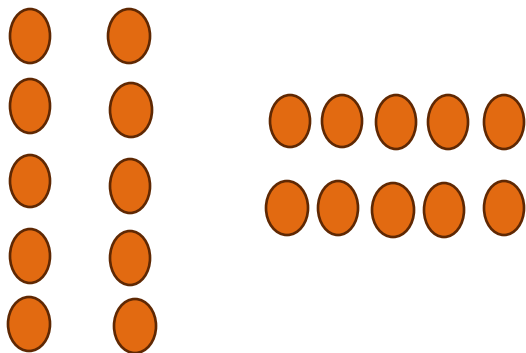
Can count in 5s



Can count in 10s



Can use concrete objects, pictorial representations and arrays



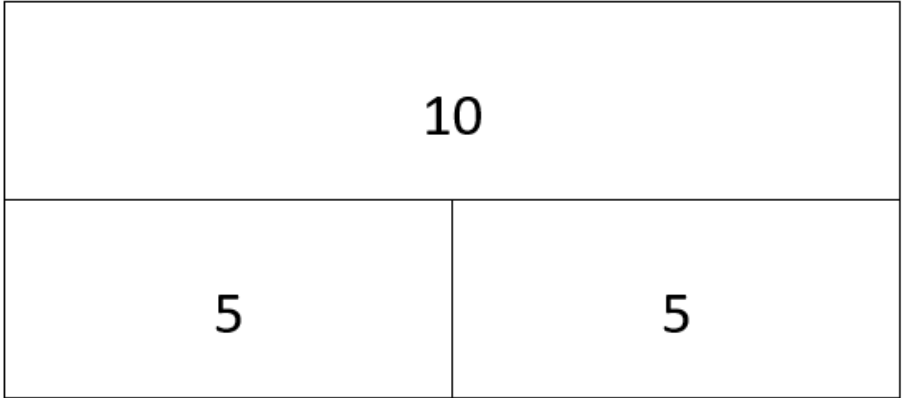
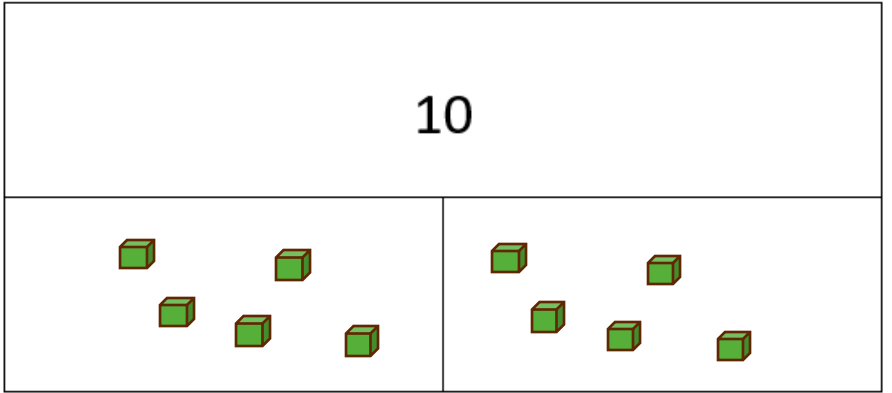
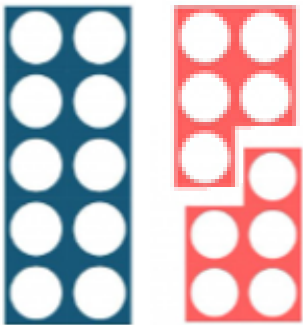
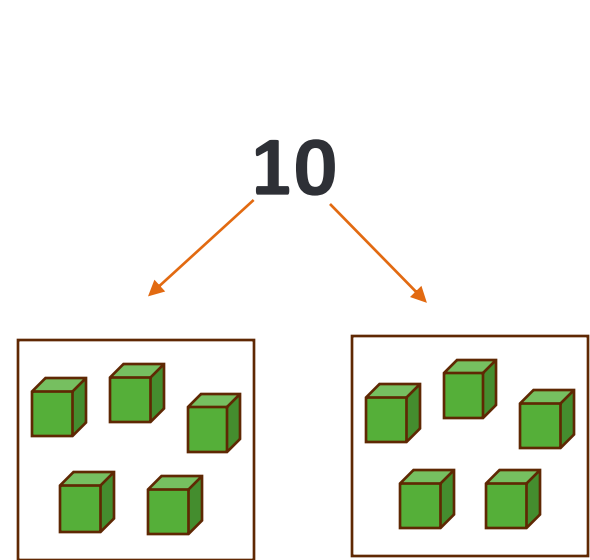
?									
●	●	●	●	●	●	●	●	●	●

?									
●	●	●	●	●	●	●	●	●	●

?				
2	2	2	2	2

?	
5	5

Can use concrete objects, pictorial representations and arrays

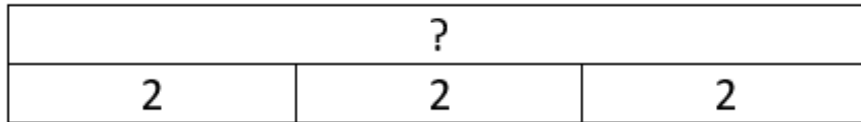
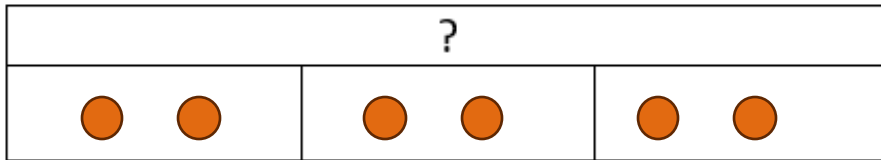
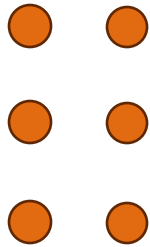


Year 2**Multiplication and Division**

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

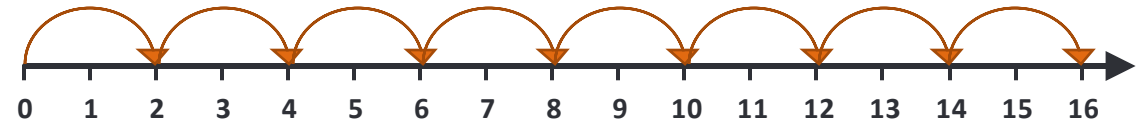
Year 2 multiplication

Can make connections between arrays, number patterns, and counting in 2s, 5s and 10s

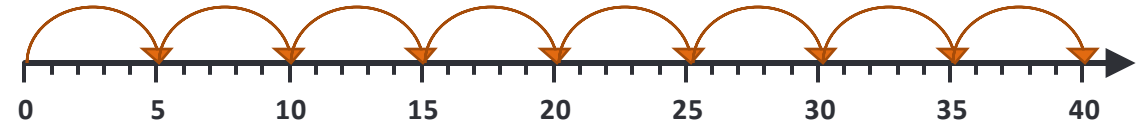


Can recall and use multiplication facts for the 2, 5 and 10 times tables

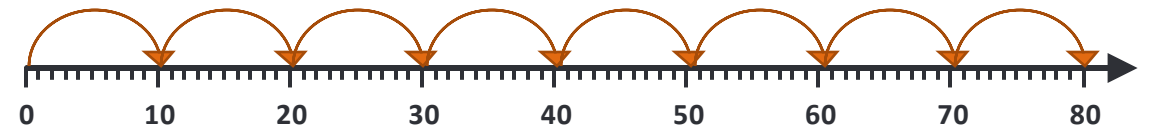
How many twos equal 16?



How many fives equal 40?

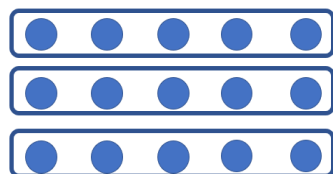


How many tens equal 80?

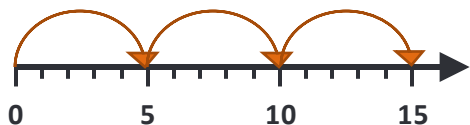


Year 2 multiplication

Can calculate mathematical statements for multiplication using the 2, 5 and 10 times tables



$$5 \times 3 = 5 + 5 + 5$$

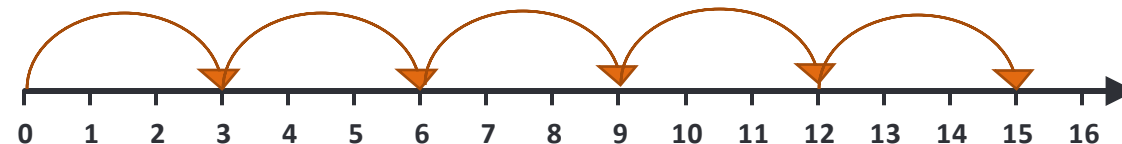
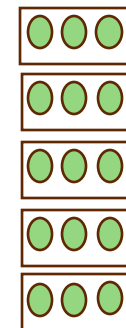


$$5 \times 3 = 15$$

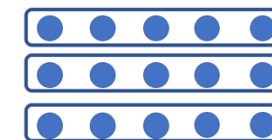
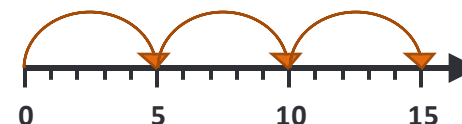
15		
5	5	5

Show that multiplication can be done in any order (commutative)

$$3 \times 5 = 15$$

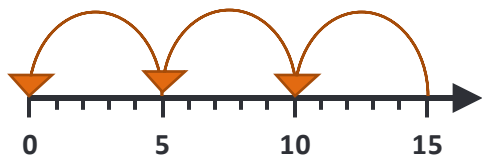
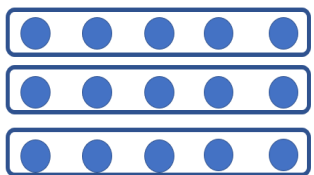


$$5 \times 3 = 15$$



Year 2 division

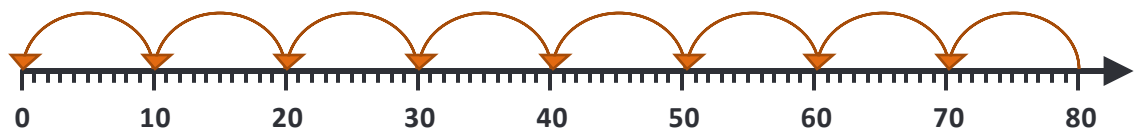
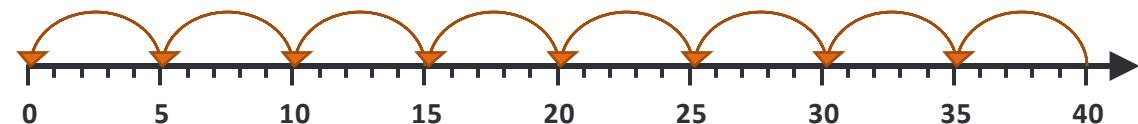
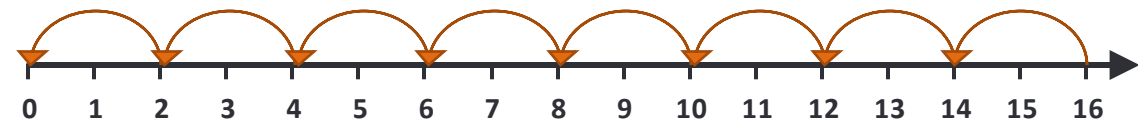
Can calculate mathematical statements for division using the 2, 5 and 10 times tables



$$15 \div 5 = 3$$

15		
5	5	5

Can recall and use division facts for the 2, 5 and 10 times tables



Year 3**Multiplication and Division**

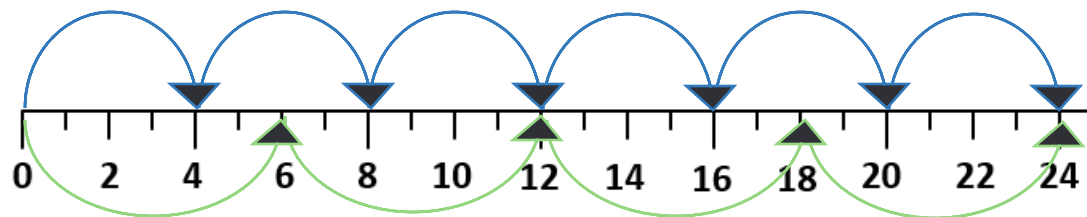
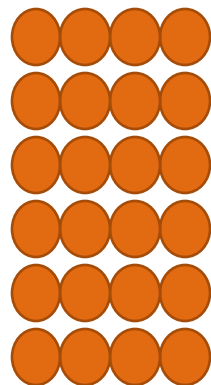
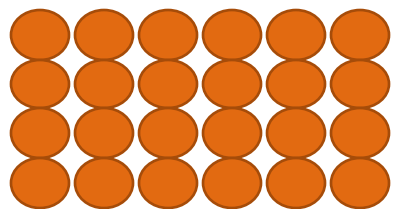
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Year 3 multiplication

Understanding commutativity

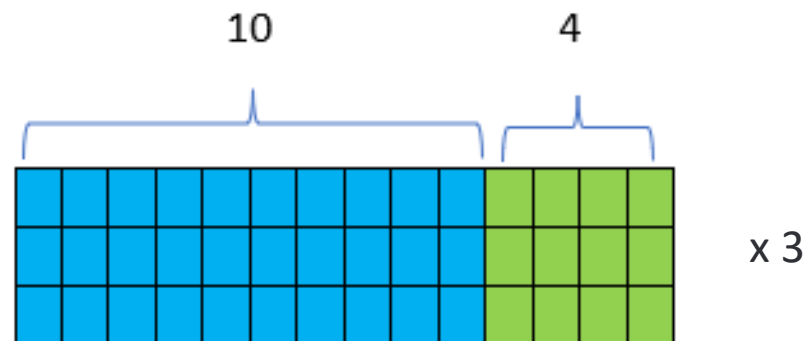
$$6 \times 4 = 24$$

$$4 \times 6 = 24$$



Can multiply 2-digit numbers by 1-digit

$$14 \times 3$$

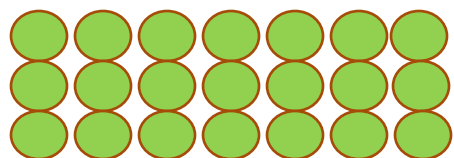


$$14 \times 3 = 10 \times 3 + 4 \times 3$$

$$14 \times 3 = 30 + 12 = 42$$

Year 3 multiplication and division

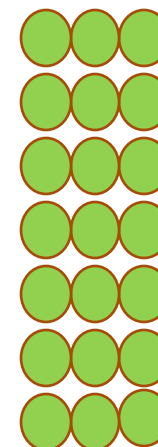
Can write mathematical statements for known multiplication and division facts using x, ÷ and =



$$7 \times 3 = 21$$

$$3 \times 7 = 21$$

21						
3	3	3	3	3	3	3



21		
7	7	7

$$21 \div 7 = 3$$

$$21 \div 3 = 7$$

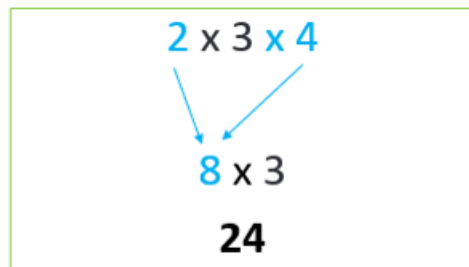
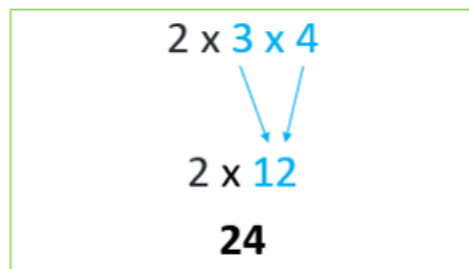
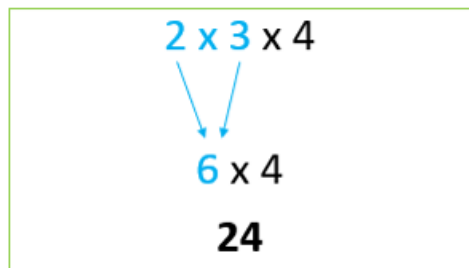
Year 4**Multiplication and Division**

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Year 4 multiplication

Can multiply 3 numbers mentally
(associative law)

$$2 \times 3 \times 4$$



Can use the distributive law to multiply 2-digit numbers by a
1-digit number

$$6 \times 32$$



$$6 \times 32 = 6 \times 30 + 6 \times 2$$

$$6 \times 32 = 180 + 12$$

$$6 \times 32 = 192$$

Year 4 multiplication

Can multiply a 2-digit number by 1-digit number (no regrouping)

$$32 \times 3$$

	3	2
×		3
		6
+	9	0
	9	6

2×3
 30×3

	3	2
×		3
	9	6

Can multiply a 2-digit number by 1-digit number (regroup)

$$54 \times 3$$

		5	4
×			3
		1	2
+	1	5	0
	1	6	2

4×3
 50×3

	5	4
×		3
1	6	2

1

Can multiply a 3-digit number by 1-digit number (regroup)

$$138 \times 4$$

	1	3	8
×			4
		3	2
+	1	2	0
+	4	0	0
	5	5	2

8×4
 30×4
 100×4

	1	3	8
×			4
5	5	2	

3

Year 4 division

Can divide a 2-digit number by 1-digit (no regrouping)

69 ÷ 3

tens	ones
10 10	1 1 1
10 10	1 1 1
10 10	1 1 1

	2	3	
3	6	9	

Can divide a 2-digit number by 1-digit (with regrouping)

72 ÷ 3

tens	ones
10 10	1
10 10	1
10 10	

→

tens	ones
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1

10 → 1 1 1 1 1 1 1 1 1 1

	2	3	
3	7	2	

Can divide a 3-digit number by 1-digit

476 ÷ 7

		6	8
7	4	7	6

Year 5

Multiplication and Division

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

Year 5 multiplication

Can multiply a 4-digit number by 1-digit number using short multiplication

$$2,169 \times 8$$

		2	1	6	9
×					8
	1	7	3	5	2
	1	1	5	7	

Can multiply a 2-digit number by a 2-digit number using long multiplication

$$23 \times 46$$

			2	3
×			4	6
		1 ₁	3 ₁	8
+		9 ₁	2	0
	1	0	5	8

Can multiply a 3-digit number by a 2-digit number using long multiplication

$$254 \times 37$$

			2	5	4
×				3	7
		1 ₁	7 ₃	7 ₂	8
+		7 ₁	6 ₁	2	0
		9	3	9	8

Can multiply a 4-digit number by a 2-digit number using long multiplication

$$1,836 \times 24$$

			1	8	3	6
×					2	4
			7 ₃	3 ₁	4 ₂	4
+		3 ₁	6	7 ₁	2	0
		4	4	0	6	4

Year 5 division

Can divide a 4-digit number by a 1-digit number
Short division (no remainder)

$$8,034 \div 6$$

	1	3	3	9
6	8	² 0	² 3	⁵ 4

Can divide a 4-digit number by a 1-digit number
Short division (with remainder)

$$6,287 \div 5$$

	1	2	5	7	r 2
5	6	¹ 2	² 8	³ 7	

Year 6**Multiplication and Division**

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving addition, subtraction, multiplication and division
- multiply one-digit number with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places

Year 6**Fractions (including decimals and percentages)**

- multiply one-digit number with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places $\frac{3}{8}$

Year 6 multiplication

Can multiply up to 4 digits by a two-digit whole number using the formal written method of long multiplication

4,132 x 24

		4	1	3	2
×				2	4
	1	6	5 ₁	2	8
+	8	2	6	4	0
	9	9	1	6	8
		1			

Can multiply a one-digit number with up to two decimal places by whole numbers

2.14 x 6

	2	.	1	4
×				6
	1	2	.	8
	1		2	

Year 6 division

Can divide numbers up to 4 digits by a one-digit whole number using long division

$$496 \div 8$$

			6	2
	8	4	9	6
	-	4	8	0
			1	6
			1	6
				0

$$8 \times 60 = 480$$

$$8 \times 2 = 16$$

I know
 $8 \times 6 = 48$
so $8 \times 60 = 480$

Can divide numbers up to 4 digits by a two-digit whole number using long division

$$465 \div 15$$

			3	1
1	5	4	6	5
	-	4	5	0
			1	5
	-		1	5
				0

$$15 \times 30 = 450$$

$$15 \times 1 = 15$$

I know
 $15 \times 3 = 45$
so $15 \times 30 = 450$

Year 6 division

Can divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders or fractions

$$372 \div 16$$

			2	3	r 4
1	6	3	7	2	
	-	3	2	0	$16 \times 20 = 320$
			5	2	
	-		4	8	$16 \times 3 = 48$
				4	

$$372 \div 16 = 23 \text{ r } 4$$

or

$$372 \div 16 = 23 \frac{4}{16}$$

$$372 \div 16 = 23 \frac{1}{4}$$

Can use written division methods in cases where the answer has up to two decimal places

$$7.62 \div 6$$

	1	.	2	7
6	7	.	6	2